Stochastic modeling of Extinction coefficients for solar power applications

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January 2007

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Contents

1 INTRODUCTION

This document deals with the problem to realize a useful stochastic model of how the extinction coefficients, regarding the wave length region corresponding to the working area for silicon photovoltaic solarcells, will vary as a consequence of how different meteorological parameters undergo variations. As the weather situation is depending on stochastic variations, then also the extinction coefficients will vary stochastically. The extinction in question is defined according to:

Equation 1:

$$\tau = \exp(-Ext \cdot M)$$

Equation 2:

$$S = S_0 \cdot \tau$$

Equation 3:

$$M = \frac{h}{h_0}$$

Equation 4:

$$S_M = S \cdot \cos\beta + Sdiff$$

Where

τ: the atmospheric transmission (0 – 1)

Ext: the extinction coefficient (in the wave length region of silicon photovoltaic solarcells)

M : the **relative** atmospheric depth (i.e. the distance to pass through the atmosphere by the Sun radiation). It is related to the depth when the Sun is in zenith

h: the atmospheric depth

h_0: the atmospheric depth with $\alpha = \frac{\pi}{2}$

α: the Sun's altitude above the horizon

S: irradiance (W/m^2) after the radiation (in the wave length region of silicon photovoltaic solarcells) has passed the atmosphere in question

S_0: irradiance (W/m^2) before the radiation (in the wave length region of silicon photovoltaic solarcells) has passed the atmosphere in question

Sdiff: diffuse irradiance component against as a measuring surface (solar panel). The diffuse component is a result of atmospheric scattering and reflections against surrounding objects

S_M: effective irradiance (W/m^2) against a measuring surface (solar panel)

β: the angle between the surface normal of the measuring surface (solar panel) and the direction to Sun

Cos β could be calculated by the following expression:

Equation 5:

$$\cos\beta = \sin\alpha \cdot \cos\Omega_Z + \cos\alpha \cdot \sin\Omega_Z \cdot \cos(\theta - \Omega_S)$$

Where

Ω_Z: the normal angle of the measuring surface relative to zenith

Ω_S: the normal angle of the measuring surface relative to south

θ: the Sun's azimuth

α (the Sun's altitude above the horizon) and θ are calculated according to [1].

See Figure 1 for some illustration of the above parameters.

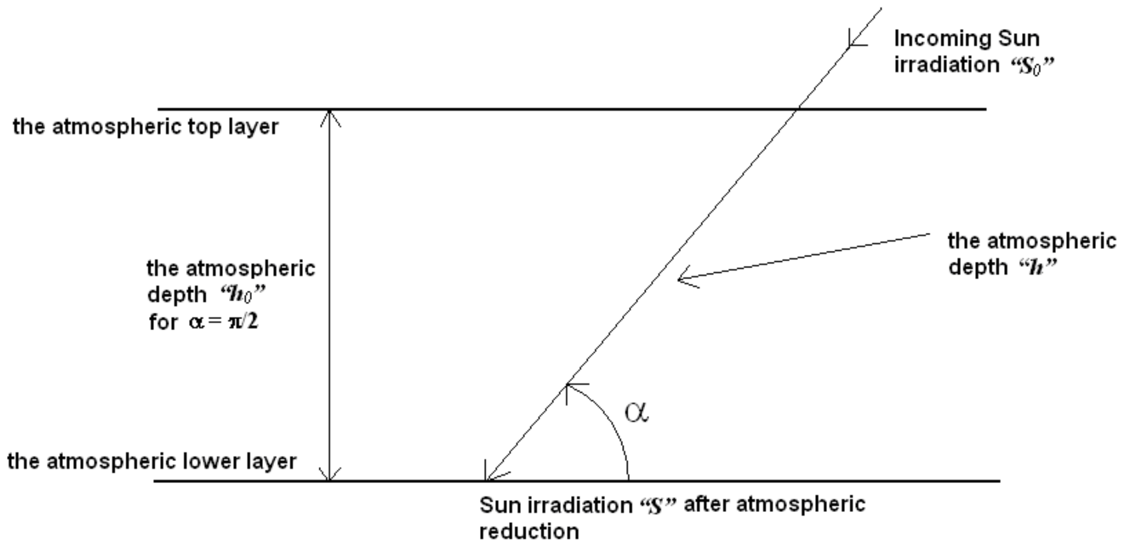


Figure 1 The radiation is reduced as an effect of the atmospheric influence

From Figure 1 it could be established the following relation between M and α:

Equation 6:

$$M = \frac{1}{\sin\alpha}$$

From **Equation 1** it could be observed that the transmission will decrease if the extinction is increasing. If the transmission is 1 ($Ext \cdot M = 0$) then there is no atmospheric reduction of the incoming Sun irradiance.

The extinction is dependent of the meteorological conditions regarding:

- Temperature
- Air pressure
- Huminiy
- Rain

- Snow
- Visibility
- Cloudness

In addition to that there is an influence on the extinction coefficient as an effect of parameters not meteorological depended:

- Varying conditions regarding aerosols in the air
- Varying conditions regarding the composition of different molecules in the air

All together there are a lot of parameters that have influences on the extinction in question.

The model that is suggested in this document, presumes so called “typical cases”, for instance:

- Ordinary summer in Sweden (a mixture of different meteorological conditions)
- Cloudy day with a clodiness of ¼
- Sunny day

And so on

Each such “typical cases” will be assigned model parameters (stochastic parameters) that serve as inputs to the model. Outputs from the model are extinction coefficients that in a statistic point of view are representative for the “typical cases” in question. See Figure 2 that shows the principle of the model that is suggested.

The reason to use “typical cases” is to get a model process that is practical to handle.

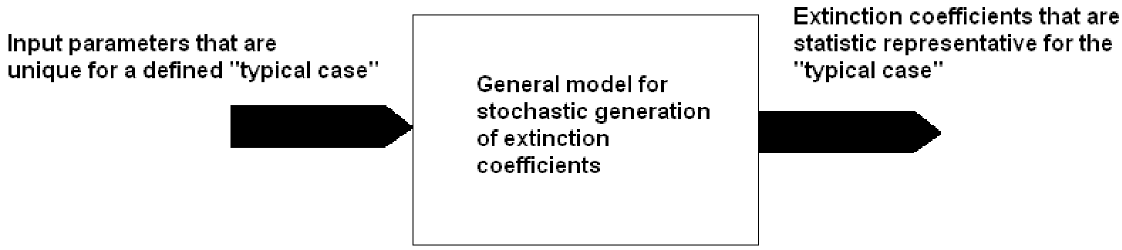


Figure 2 The principle of the stochastic model for generation of extinction coefficients

2 MODELING OF THE EXTINCTION COEFFICIENT

The principle for the model is described in [1] (chapter “Extinction_make”).

3 MEASUREMENTS

Measurements to get statistic foundations to make a survey of the extinction coefficient have been performed during the period 21/6 - 7/9 – 2006.

The principle for the measurement arrangement follows by Figure 3.

There are 3 solar cell panels connected in series by the connection box. Each solar cell panel consists of 72 series connected solar cells. The result of this arrangement is that there are 216 series connected cells at the output of the connection box. The current I_S in Figure 3 corresponds to the “short circuit current” of the solar cells. As there are so many solar cells that co-operates, two advantages are at hand:

- Small effects regarding the voltage drops in the connection wires
- A good representative value (mean value of a large number cells) regarding the short circuit current in question

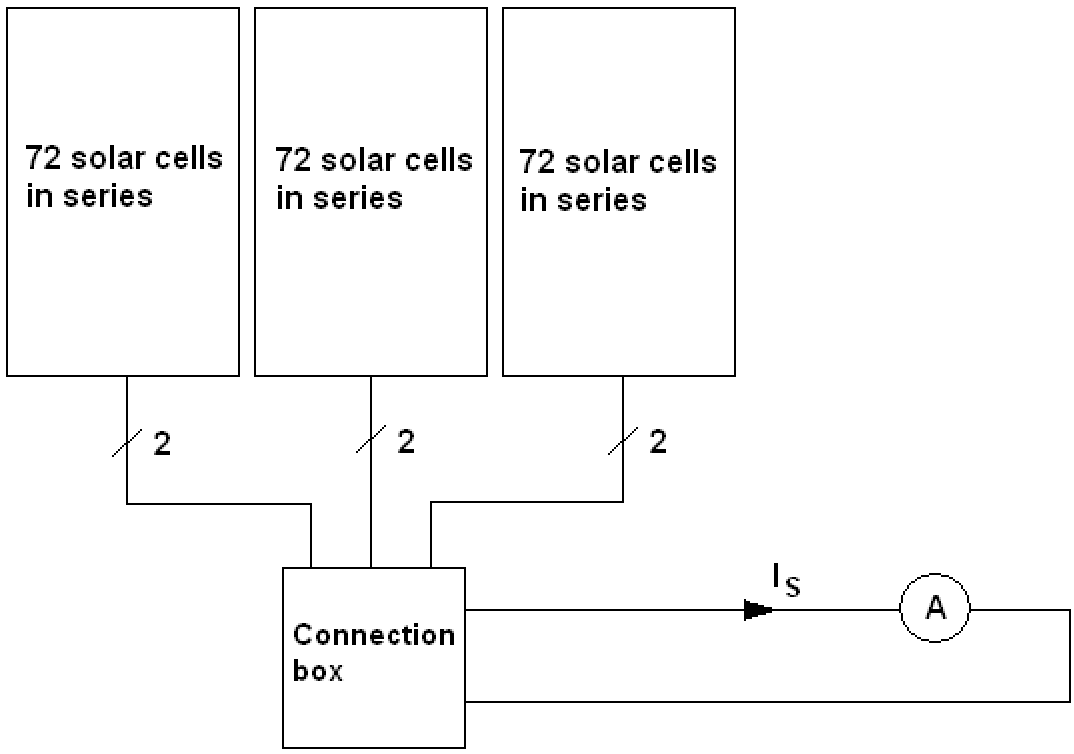


Figure 3 The principle for the measurement arrangement. The connection box connects the three solar cells panels in series. This results in 216 series connected solar cells

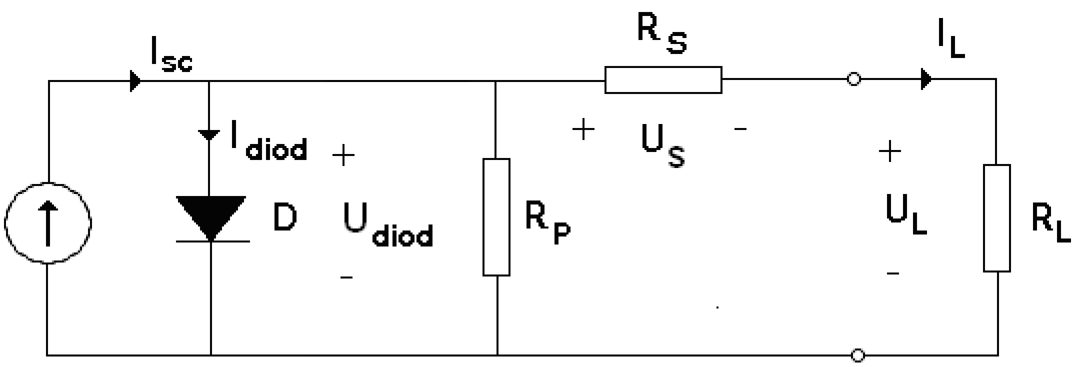


Figure 4 The equivalent circuit of a solar cell

The short circuit current is a good measure of the Sun irradiance. See Figure 4 that shows the equivalent circuit of the solar cell. As the resistance R_S in Figure 4 is quit small (about 15 mΩ) the maximum voltage drop over this resistance (i.e. at short circuit) normally is less than 60 mV. This voltage corresponds to the voltage U_S. A short circuited cell output (i.e. $U_L = 0$) will result in $U_S = U_{diod}$. If U_{diod} is in the region of maximum 60 mV, then the diod current I_{diod}, is very small (in the order of a few mA) compared with the short circuit current (normally in the order of amperes). This results in:

Equation 7:

$I_{SC} = I_L$ at short circuited cell output.

I.e.

$I_{SC} = I_S$ (see Figure 4 and Figure 3).

$I_{SC} = G \cdot S_M$,

Where

G: a scale factor (Am^2/W)

S_M: Effective Sun irradiance (W/m^2)

or

Equation 8:

$I_S = G \cdot S_M$

According to Equation 1, Equation 2 and Equation 4:

$$\tau = \exp(-Ext \cdot M)$$

$$S = S_0 \cdot \tau$$

And

$$S_M = S \cdot \cos\beta + Sdiff$$

This gives:

Equation 9:

$$I_S = G \cdot S_0 \cdot \exp(-Ext \cdot M) \cdot \cos\beta + G \cdot Sdiff = I_M \cdot \exp(-Ext \cdot M) \cdot \cos\beta + Idiff$$

Where

I_M: A reference current (A), that corresponds to the short circuit current for a Sun irradiance of S_0.

Idiff: A current component (A), that corresponds to the contribution from diffuse irradiance

I_S: Short circuited current (A)

Equation 9 gives the extinction coefficient.

Equation 10:

$$Ext = \frac{-\ln \dfrac{I_S - Idiff}{I_M \cdot \cos\beta}}{M}$$

If *Ext*, Idiff and I_M are known then it is possible to calculate a value for I_S for a given geographic position (latitude and longitude), a given normal angle of the measuring surface relative to zenith and south and a given time point (date and hour) according to:

Equation 11:

$$I_S = \exp(-Ext \cdot M) \cdot I_M \cdot \cos\beta + Idiff$$

4 MEASUREMENT RESULTS

Figure 5 to Figure 76 give the short circuit current from measurements during the period 21/6 - 7/9 – 2006. The main reason for the measurement campaign was to collect information about the short circuit current. To get some idea about the voltage variations, some days however, were used for no load voltage measuring. This result follows in Figure 77 to Figure 80.

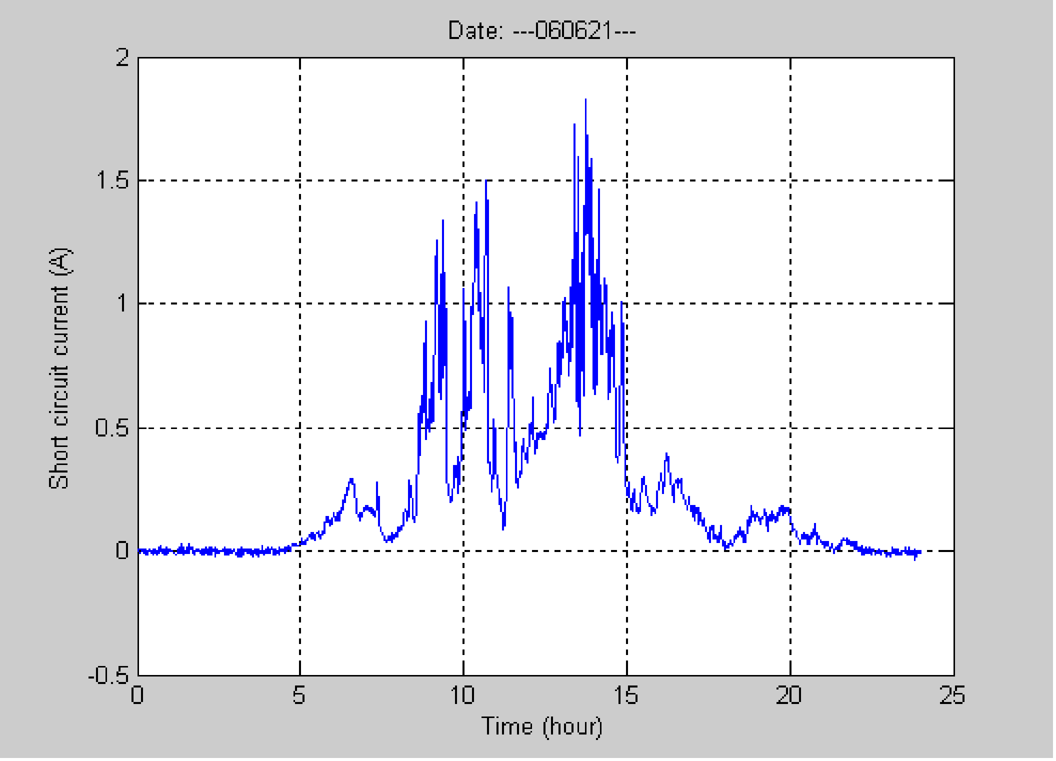


Figure 5 Short circuit current 060621

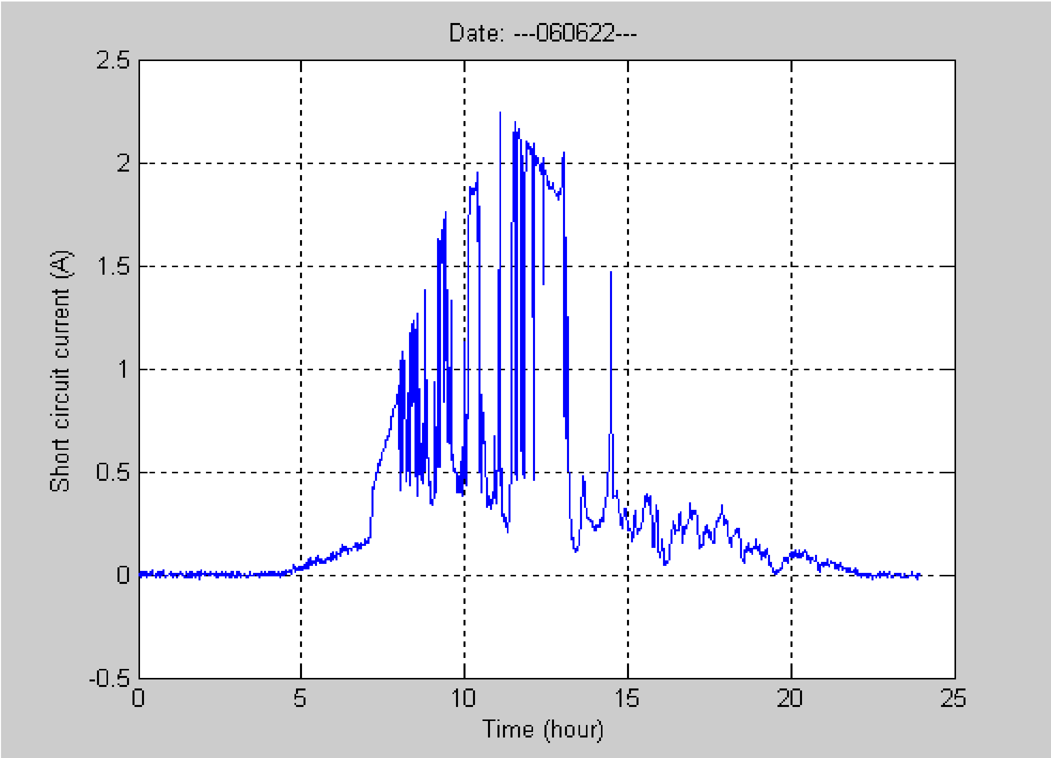
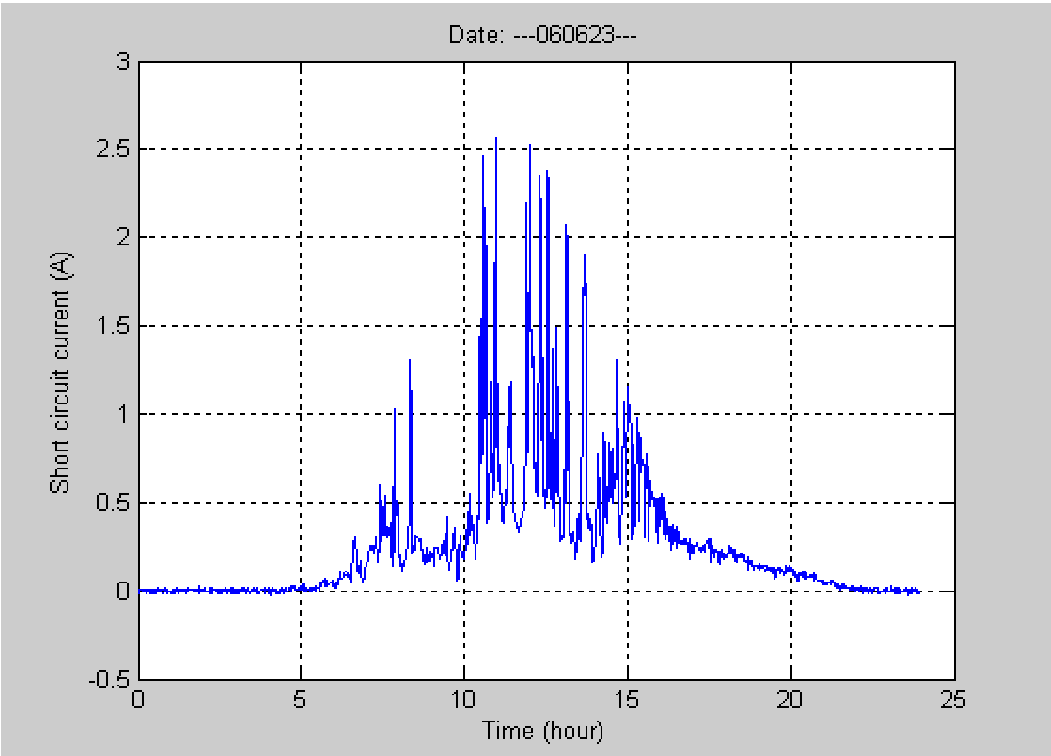


Figure 6 Short circuit current 060622



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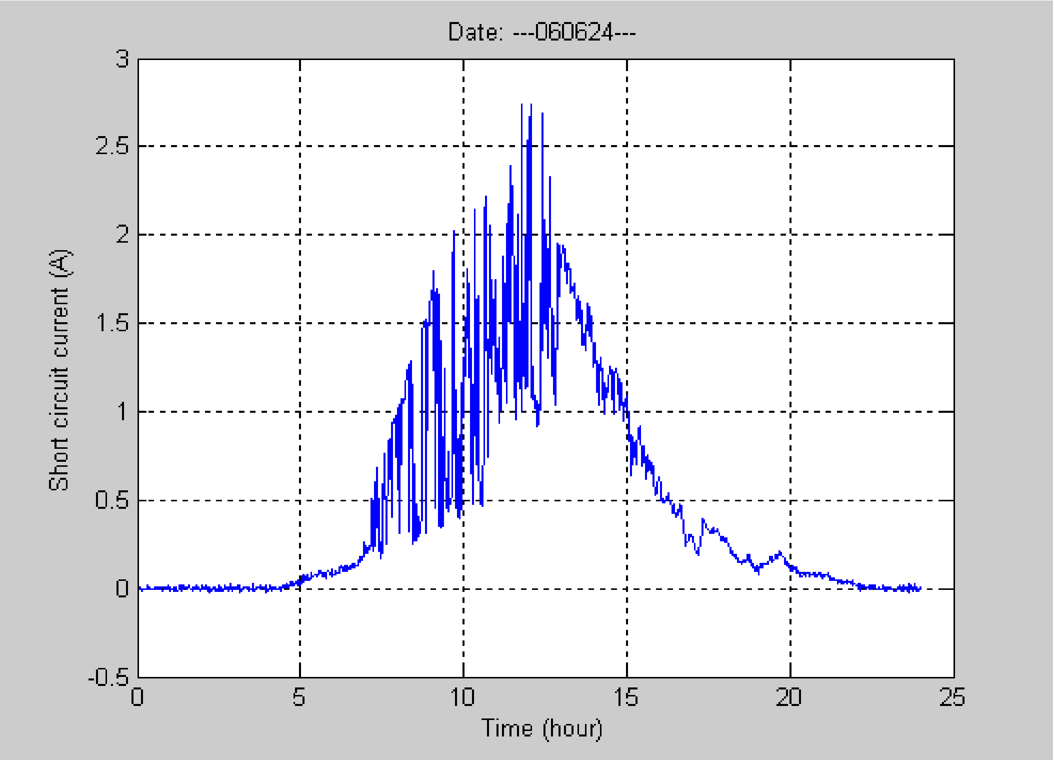

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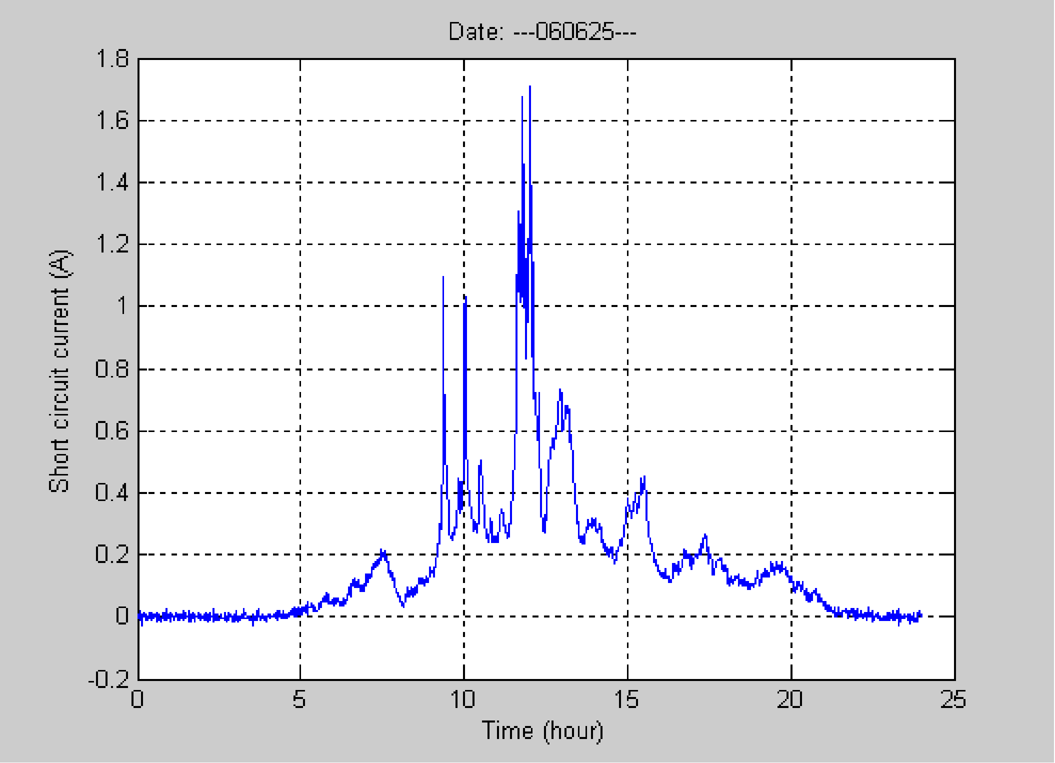


Figure 9 Short circuit current 060625

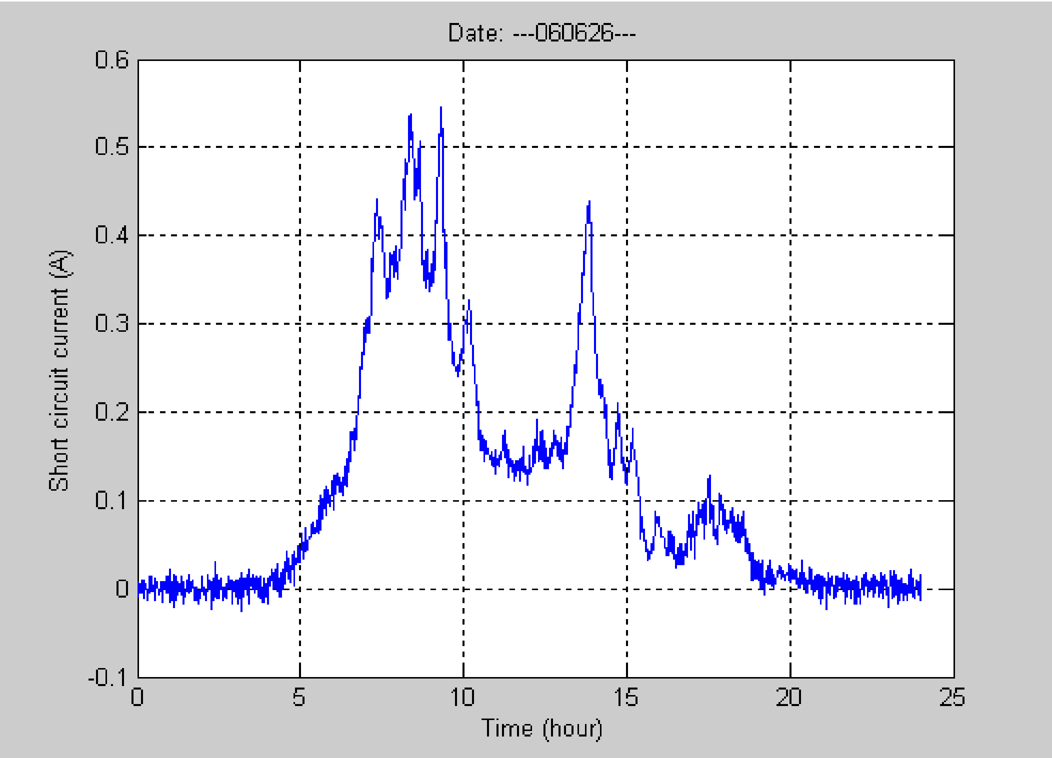


Figure 10 Short circuit current 060626

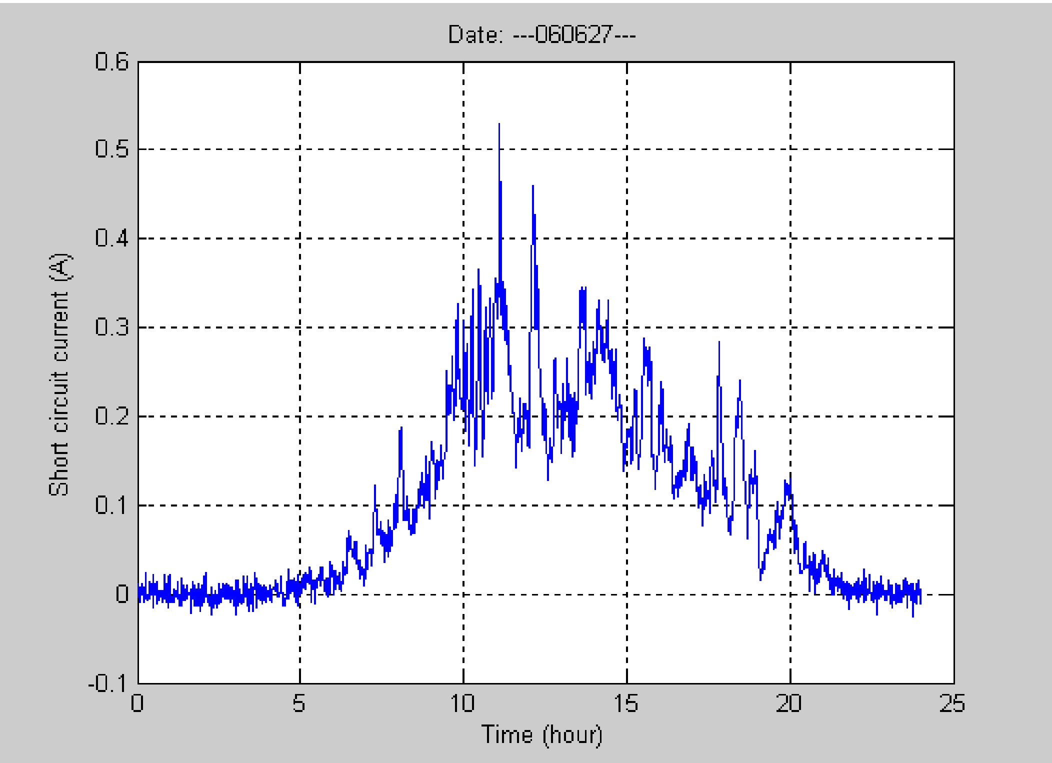


Figure 11 Short circuit current 060627

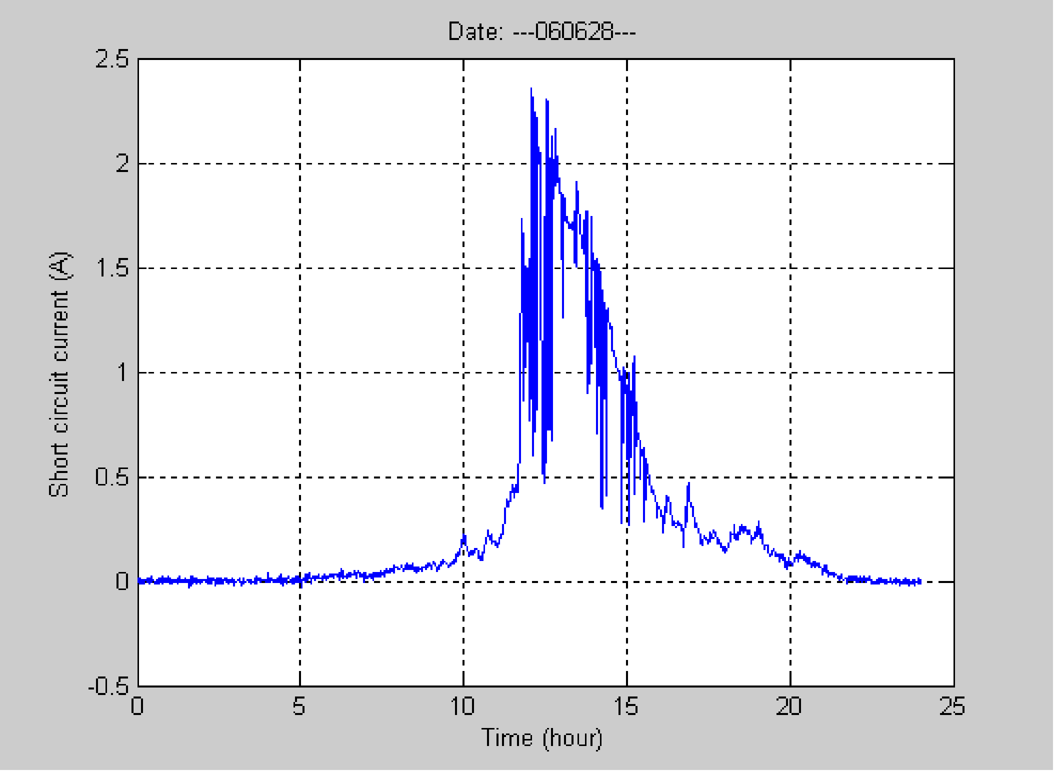


Figure 12 Short circuit current 060628

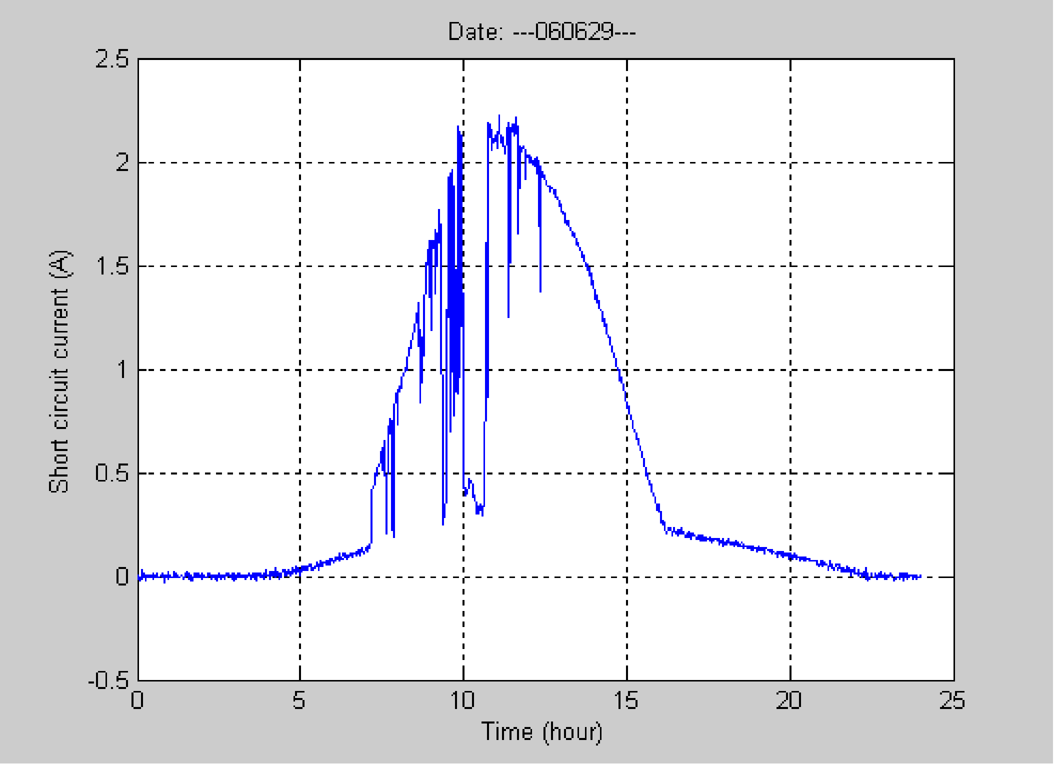


Figure 13 Short circuit current 060629

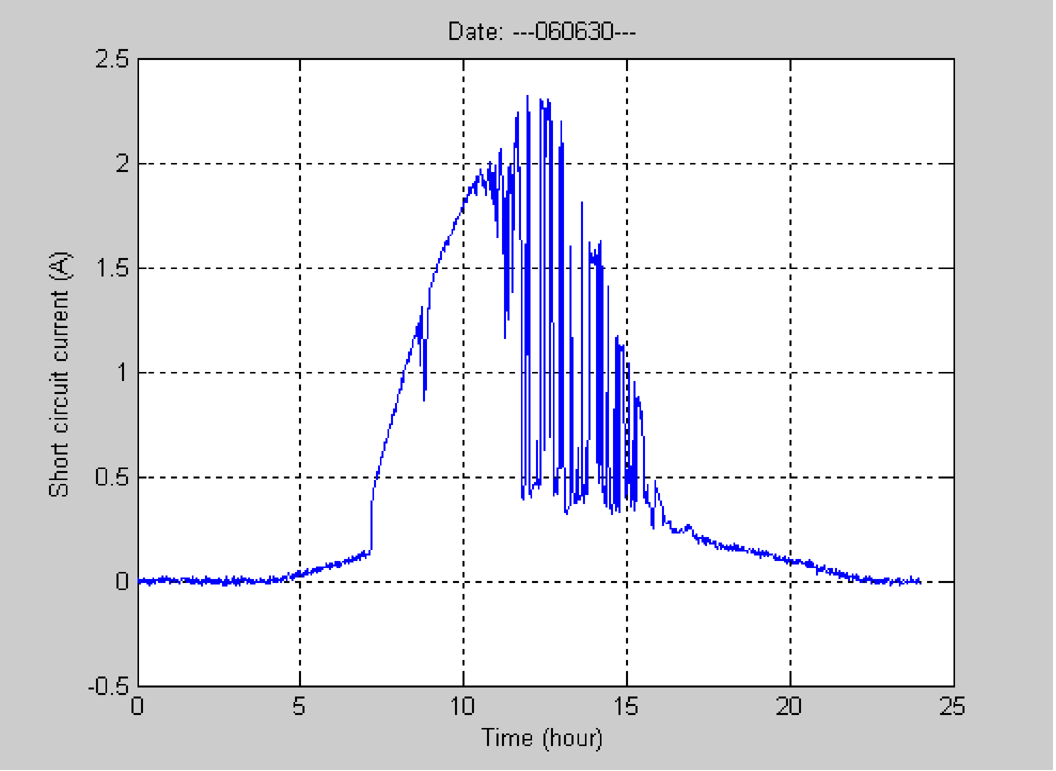


Figure 14 Short circuit current 060630

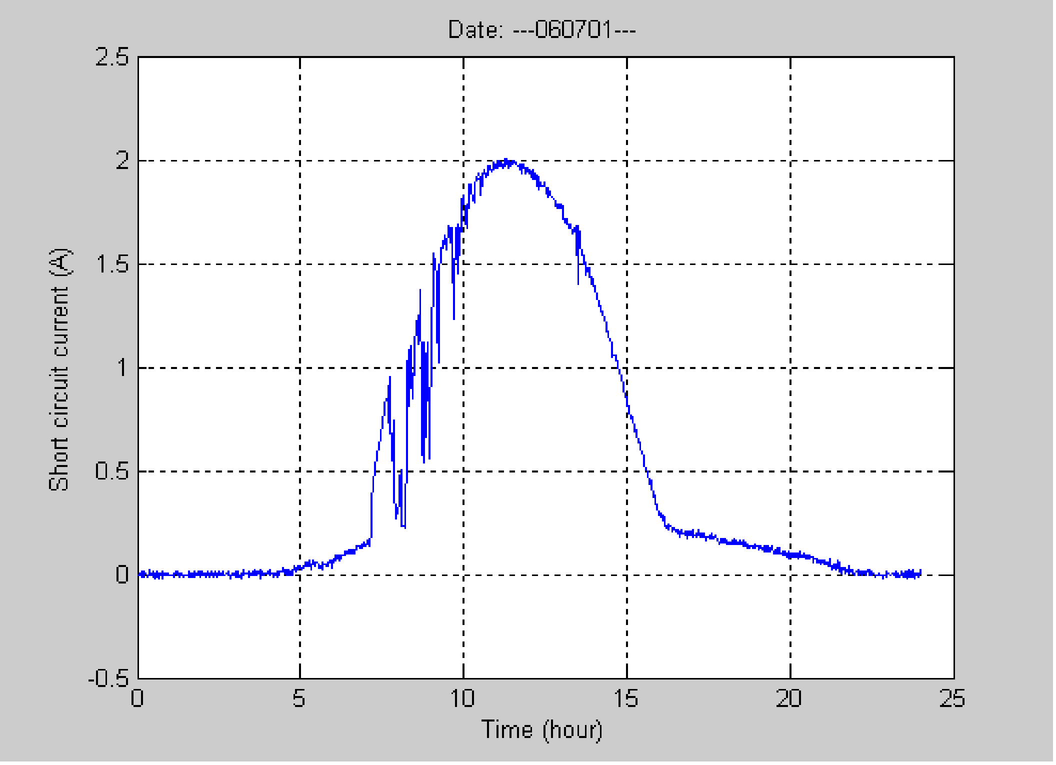
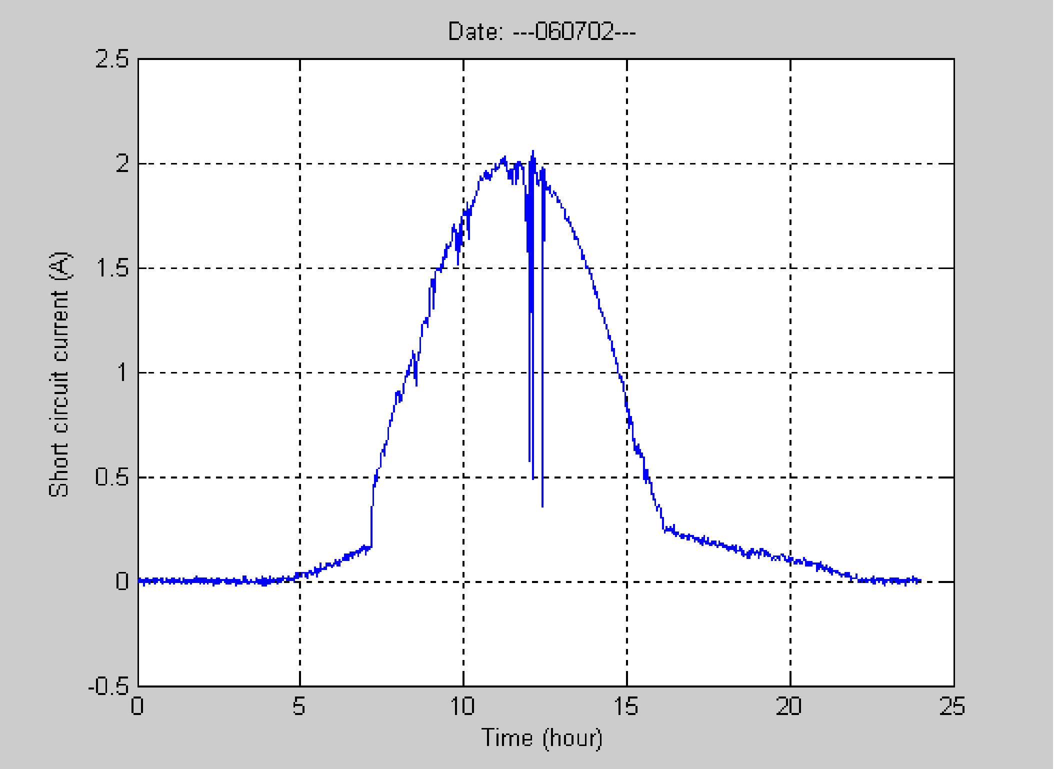


Figure 15 Short circuit current 060701



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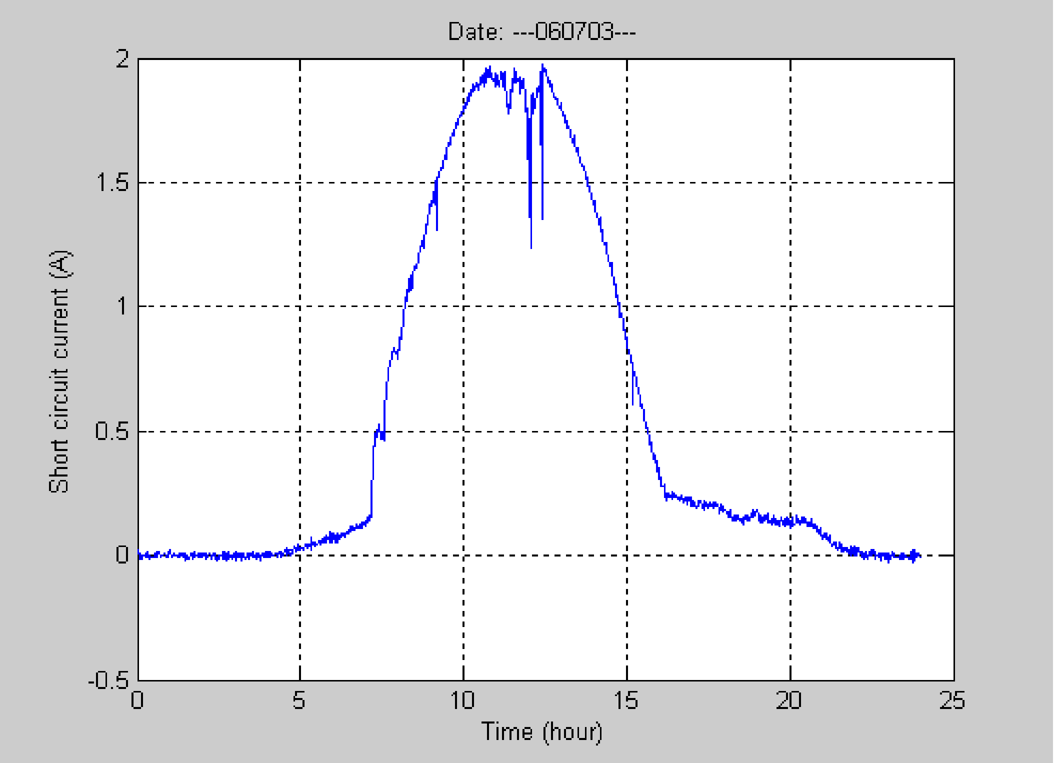

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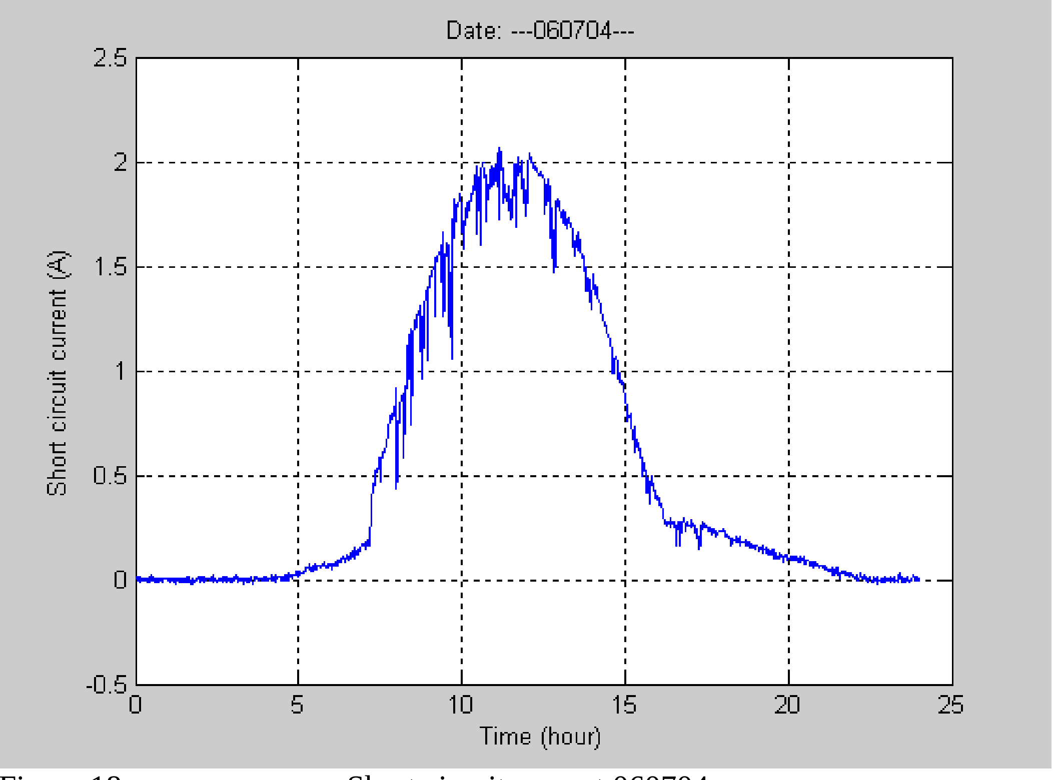

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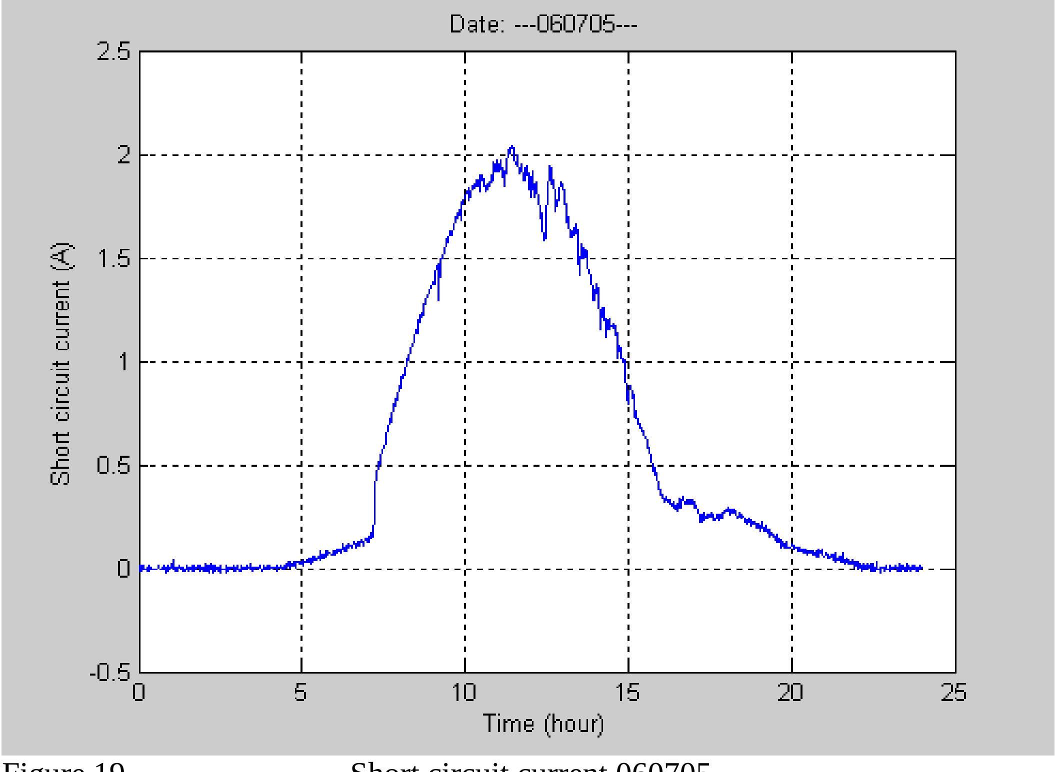

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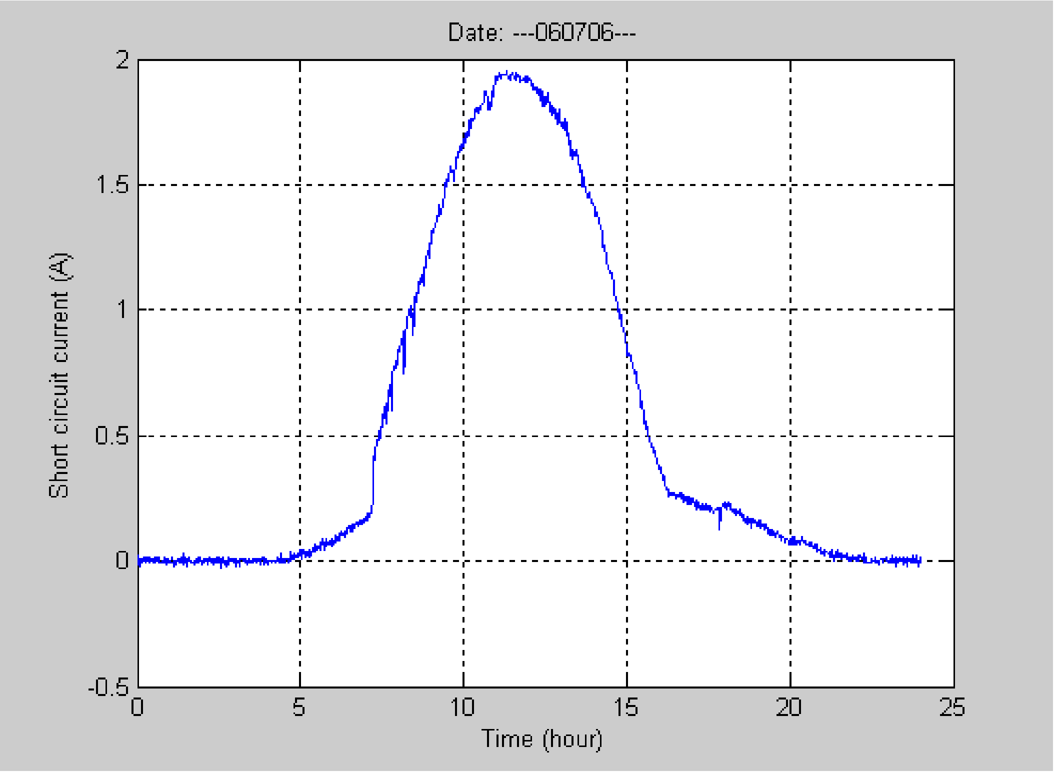

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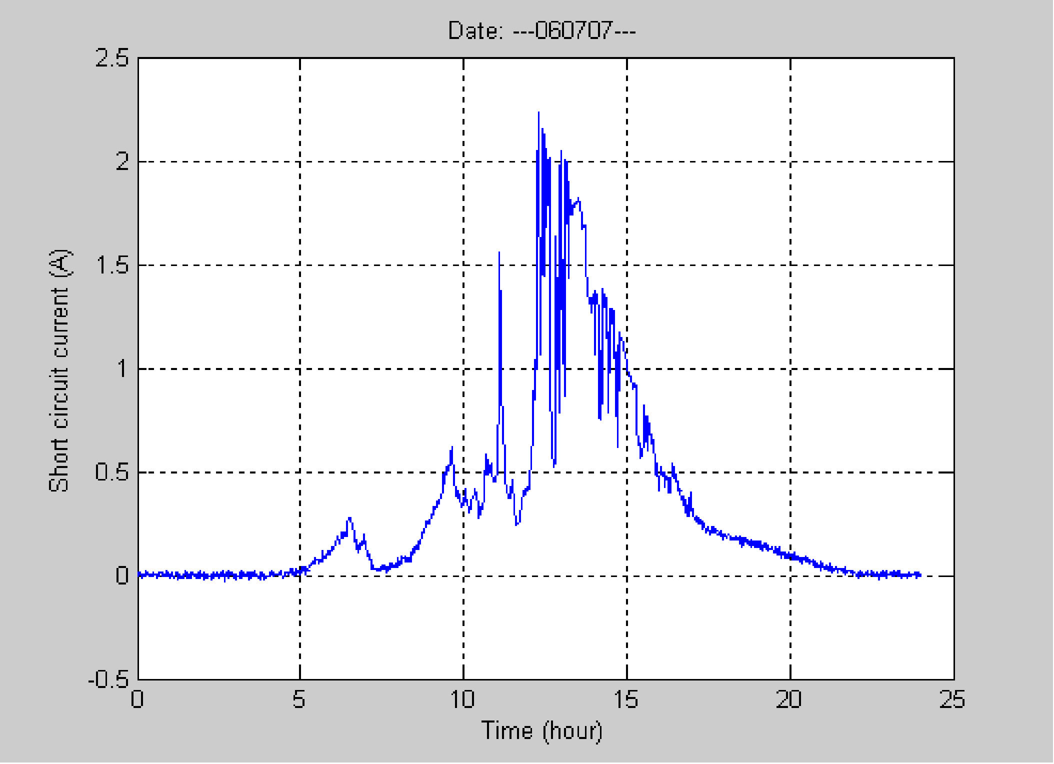

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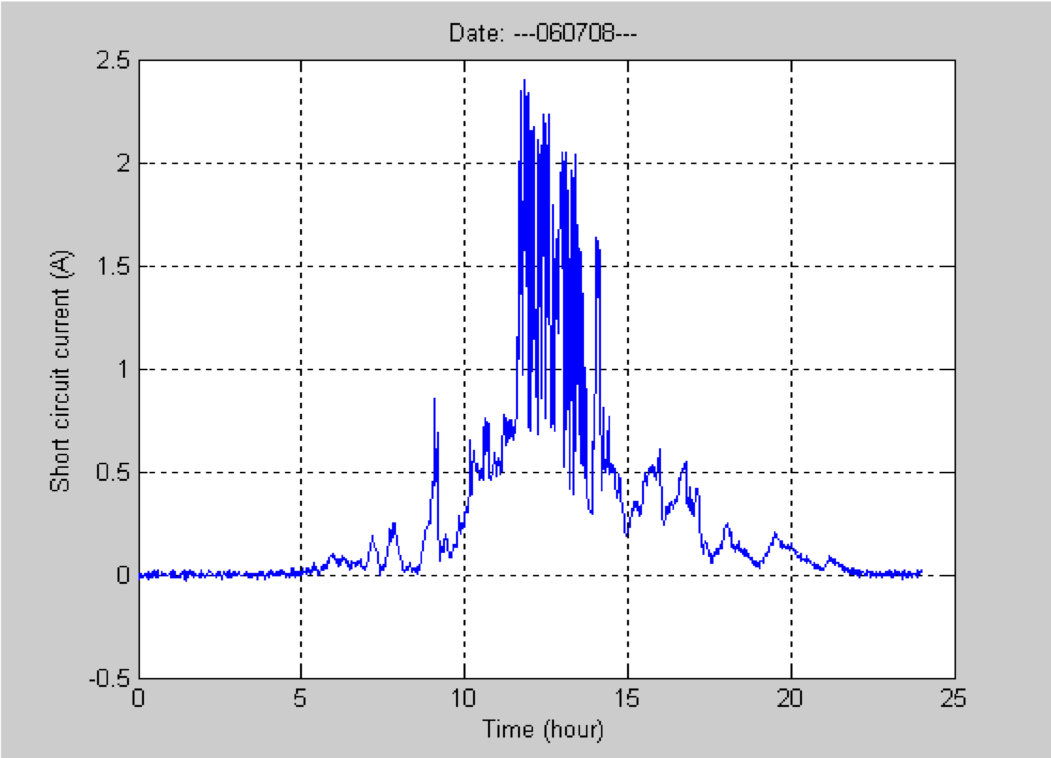

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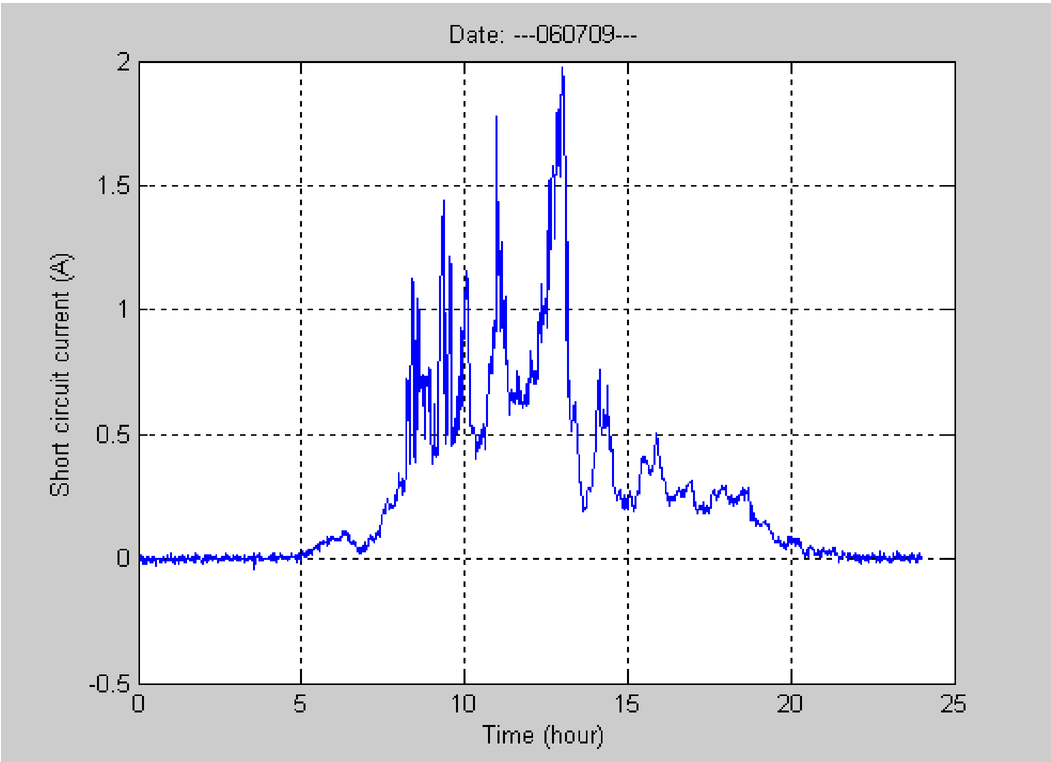

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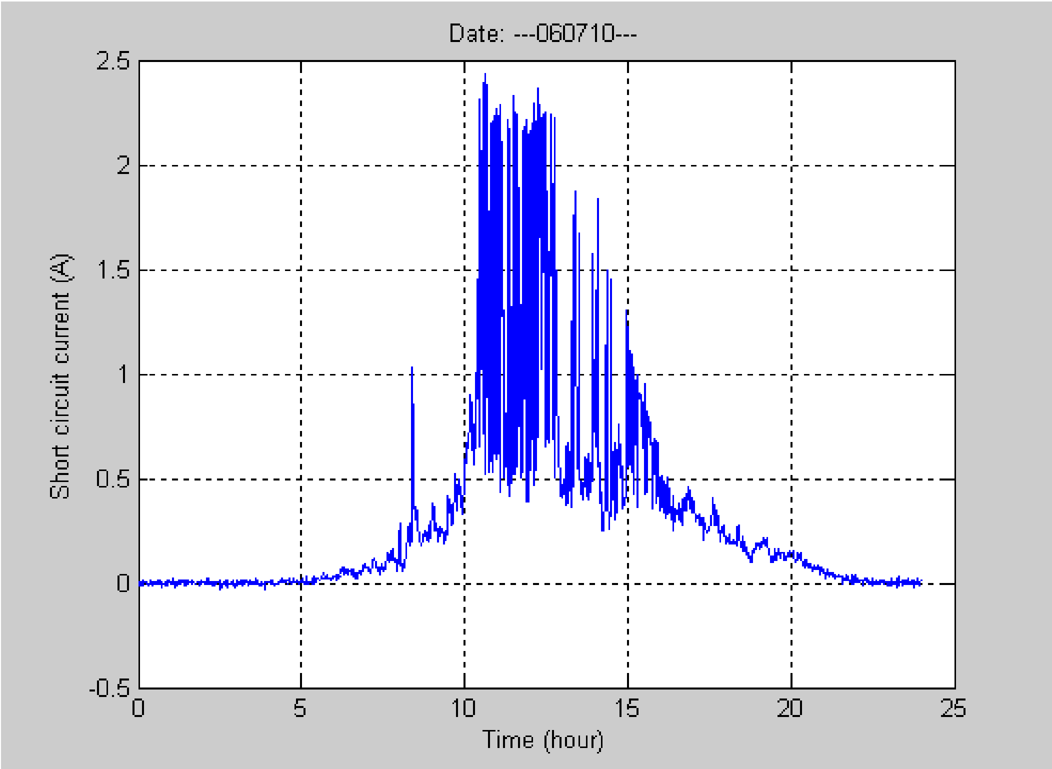

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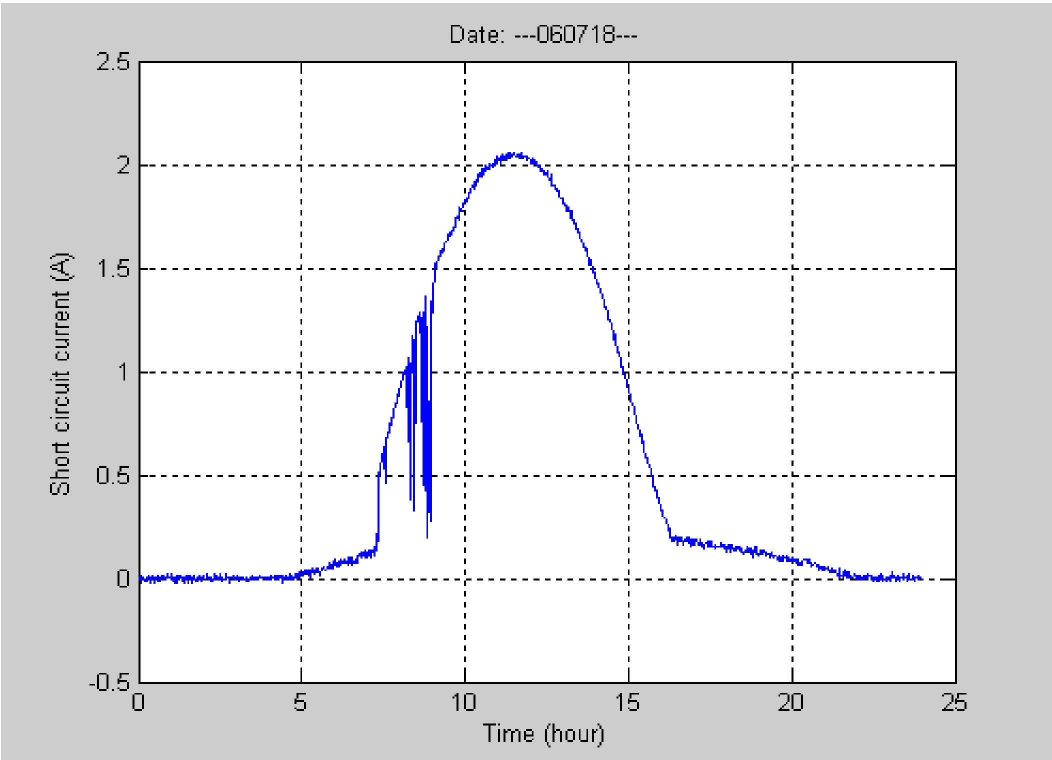

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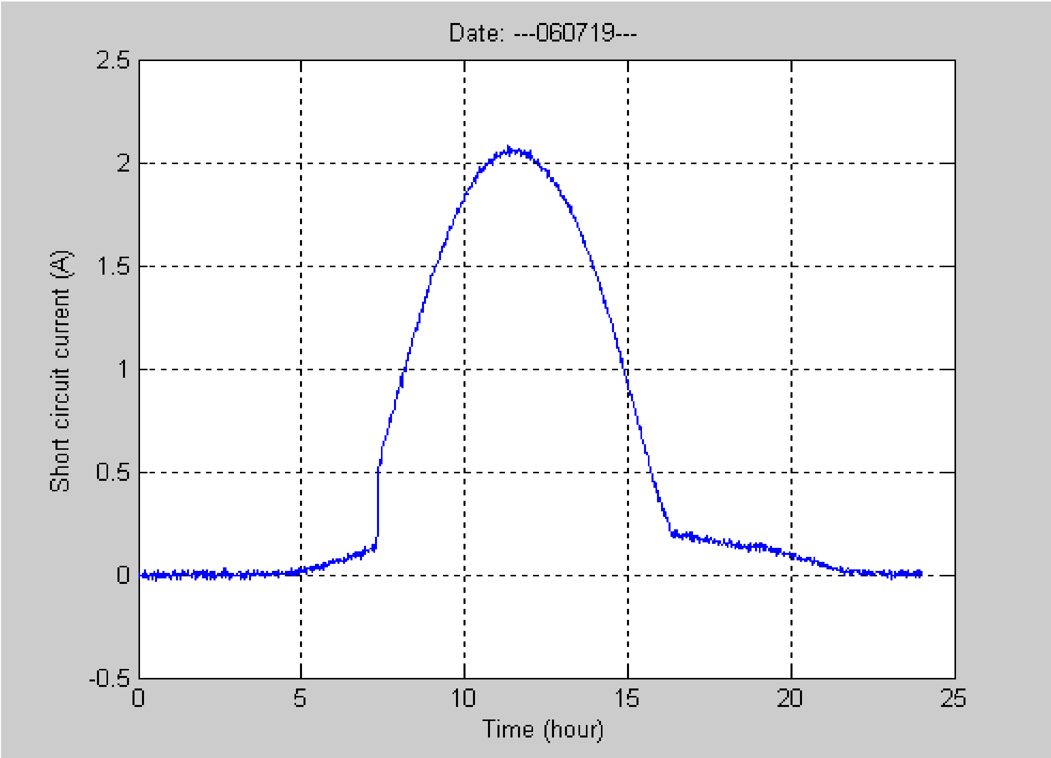

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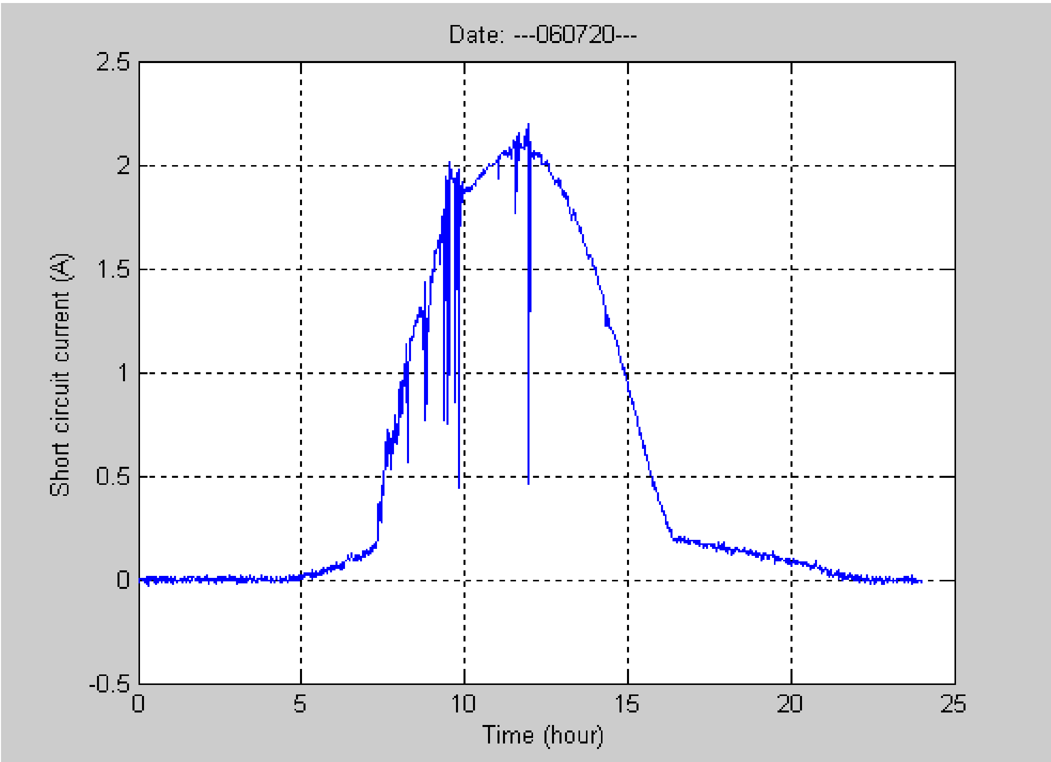

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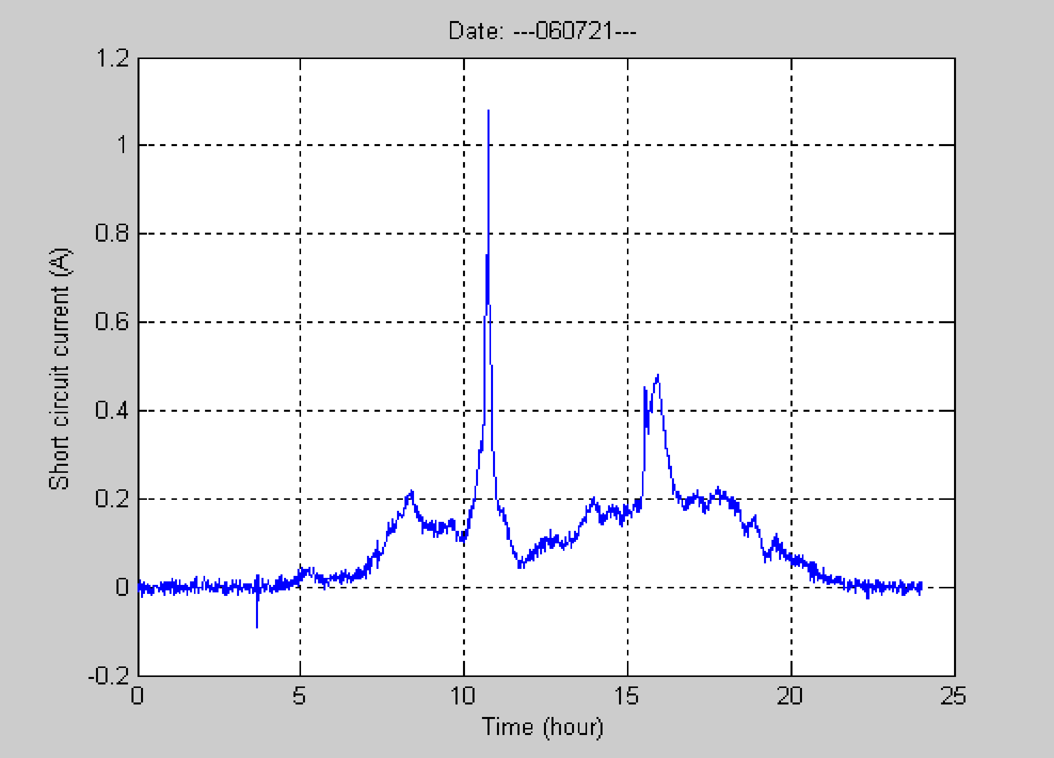

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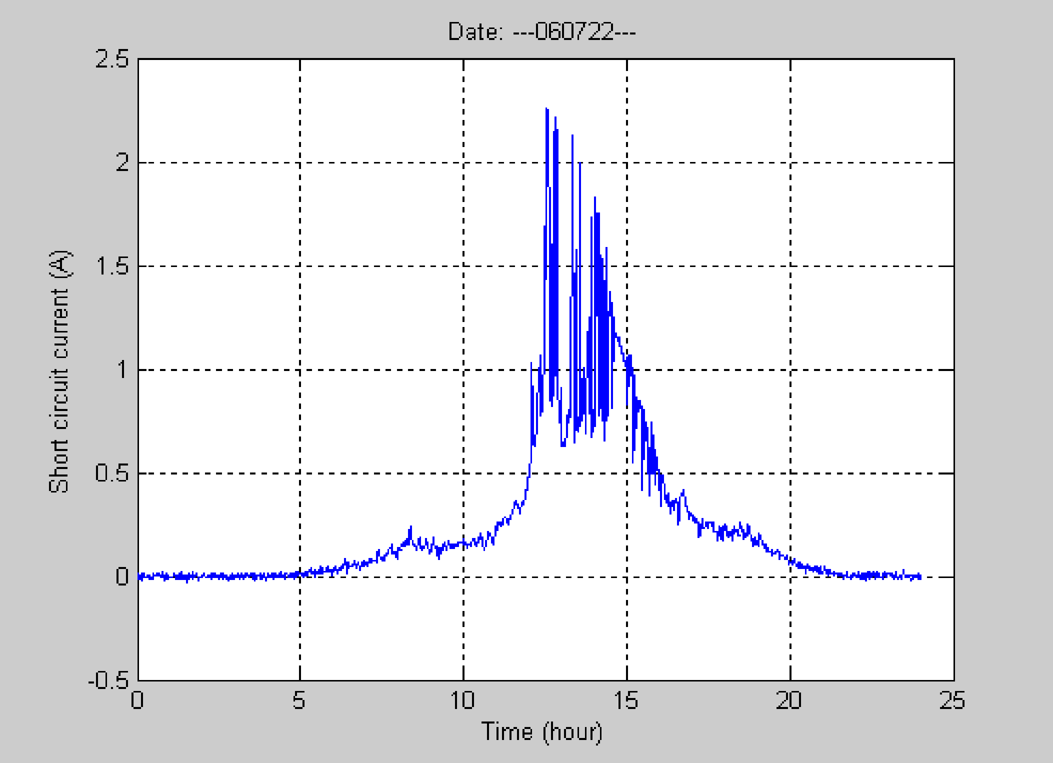

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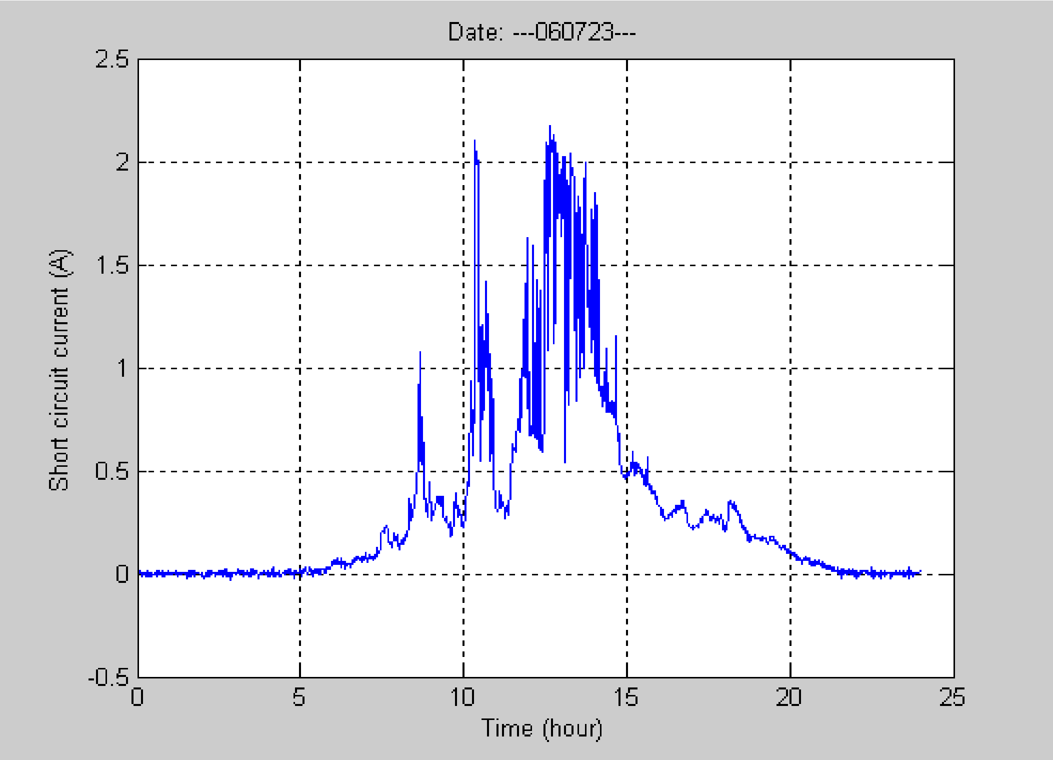

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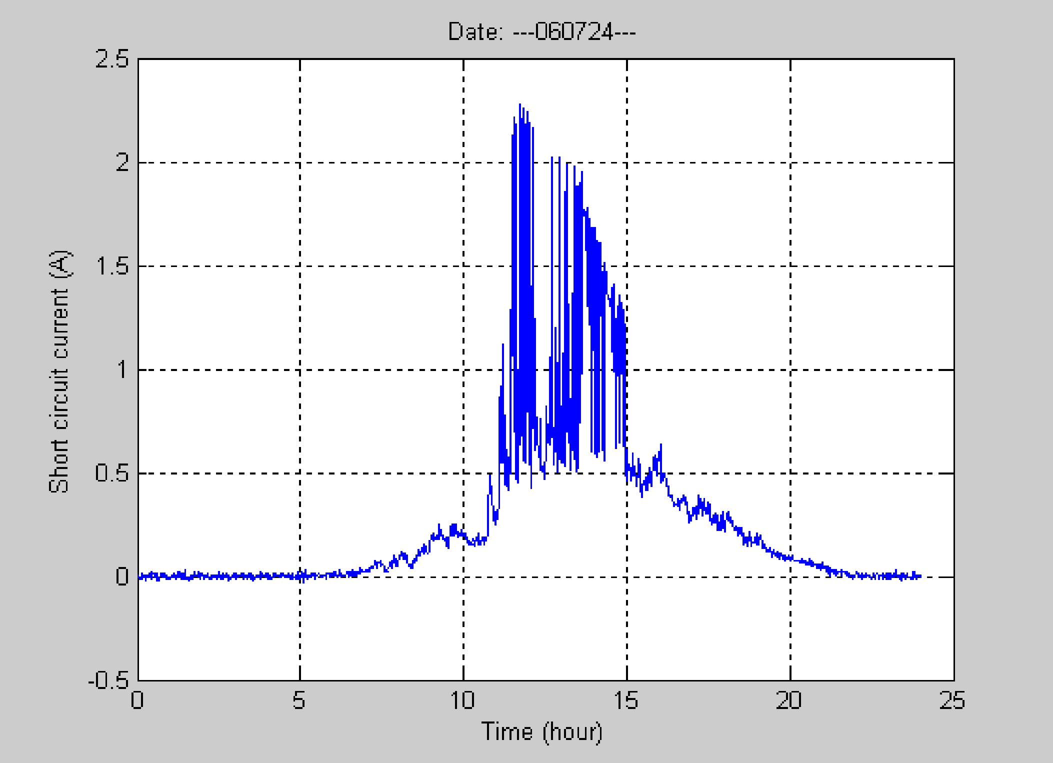

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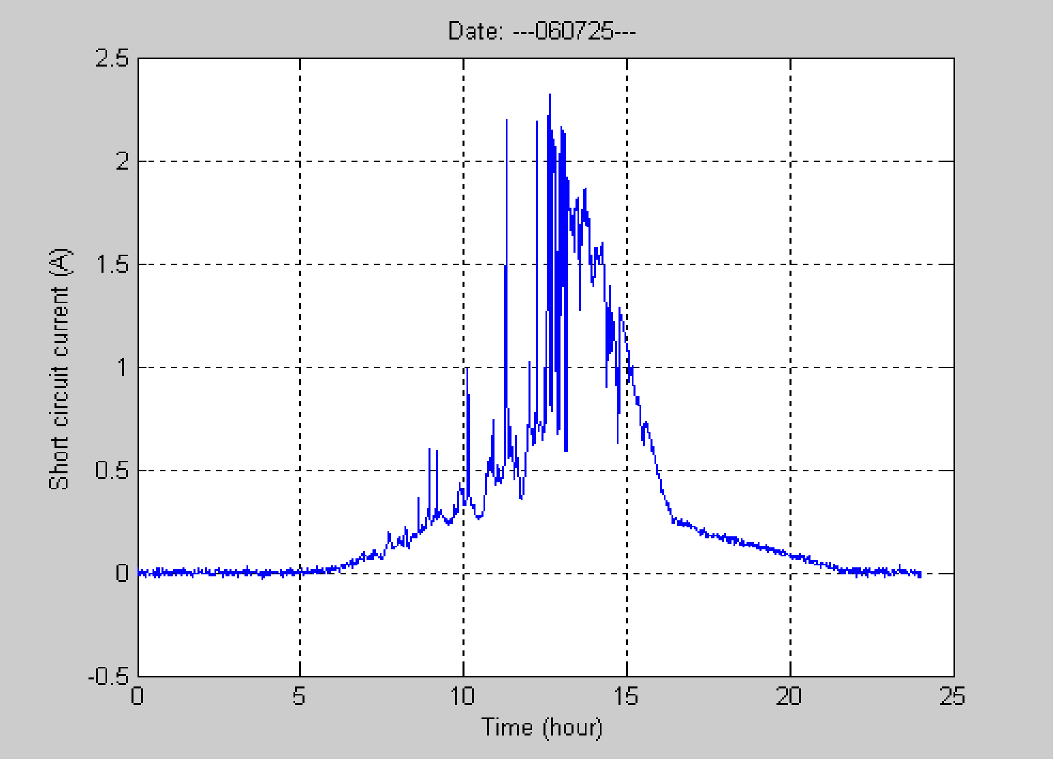

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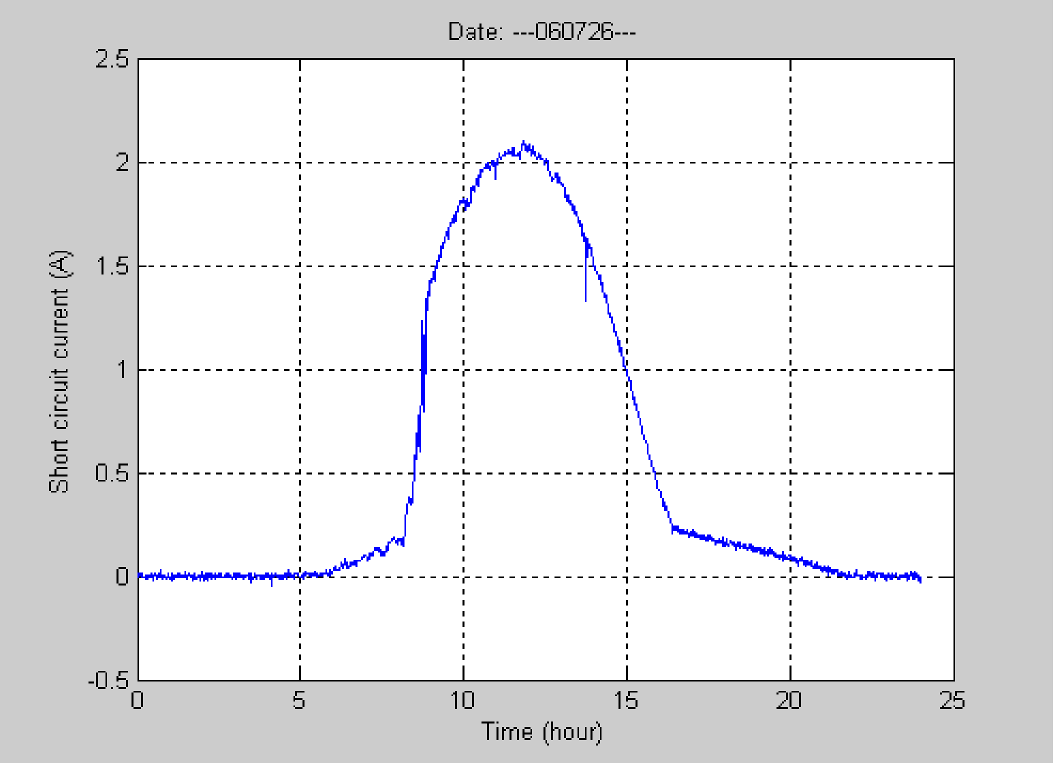

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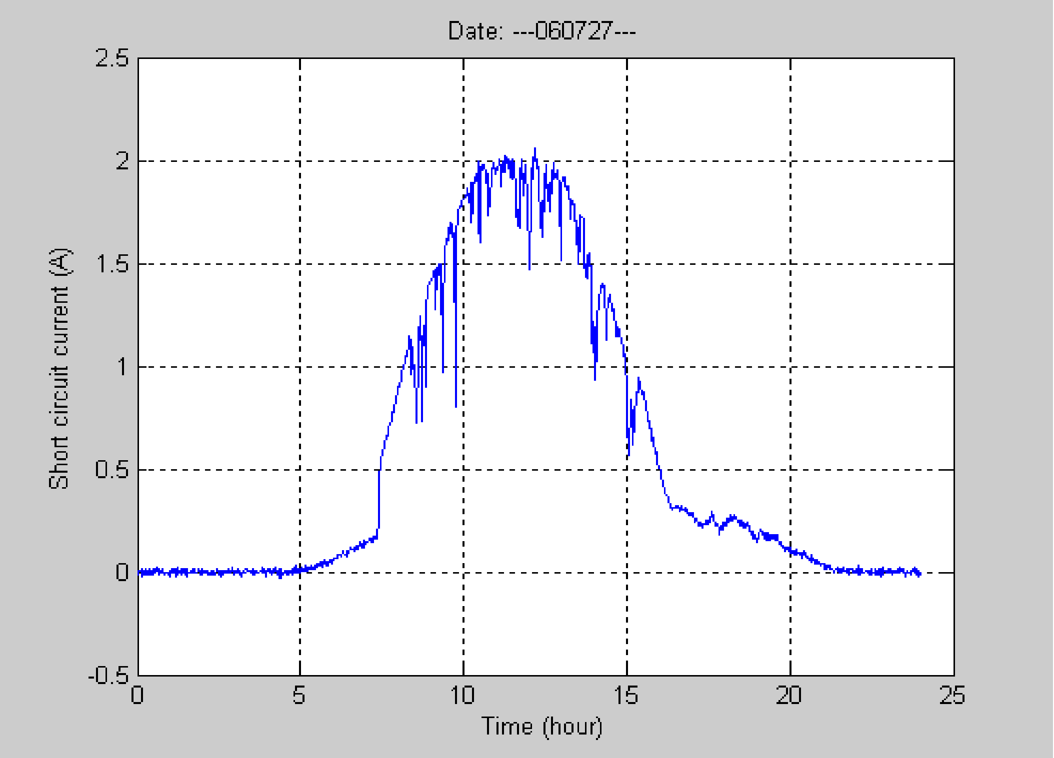

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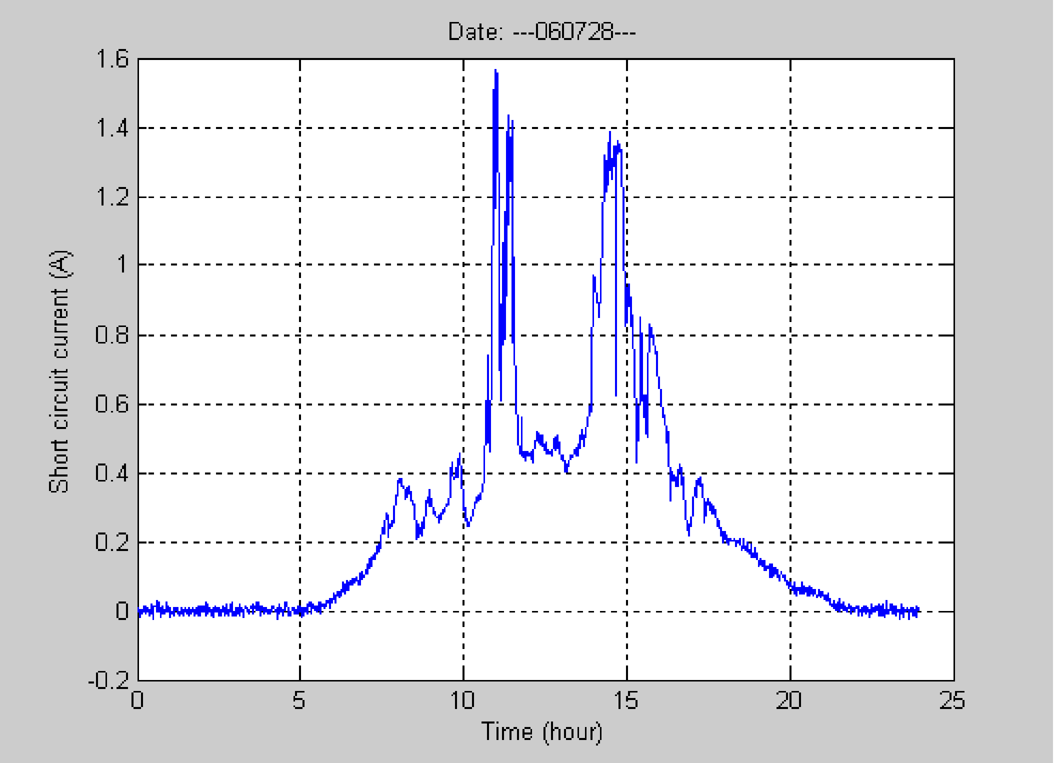

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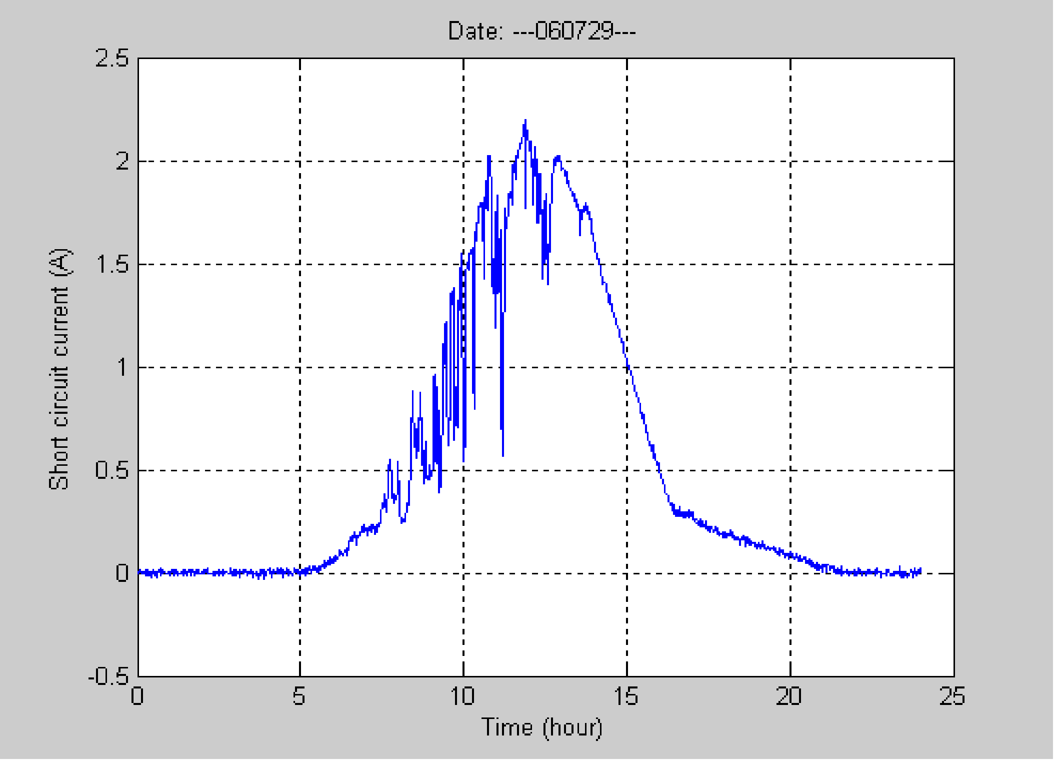

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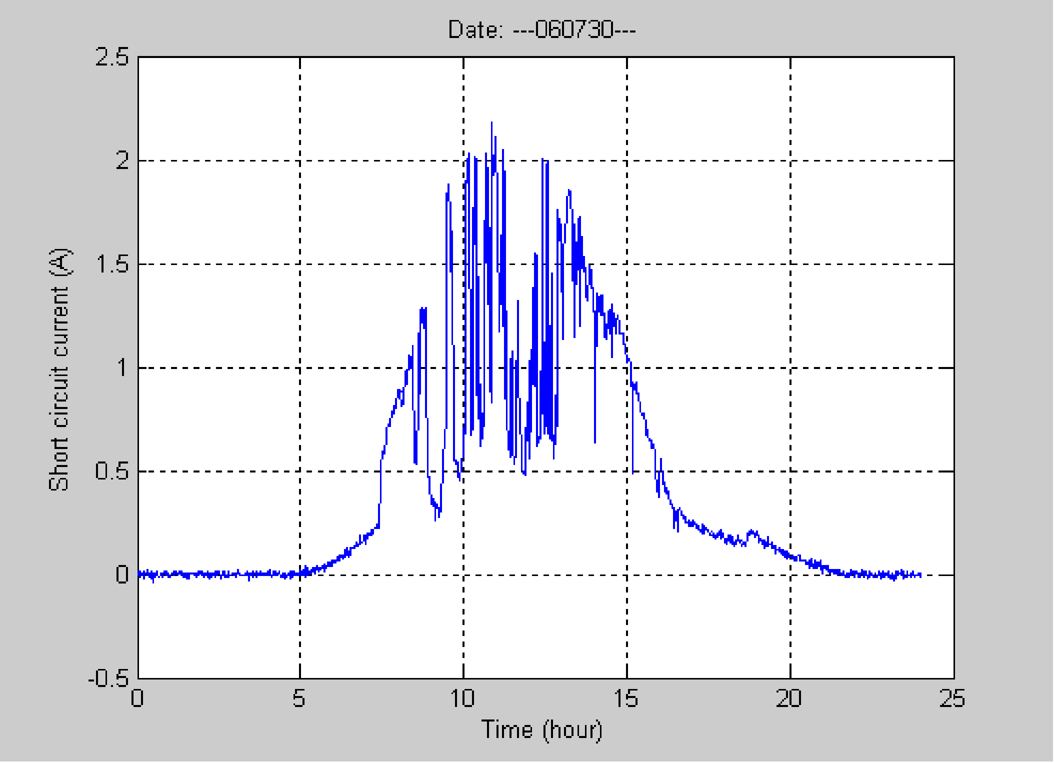

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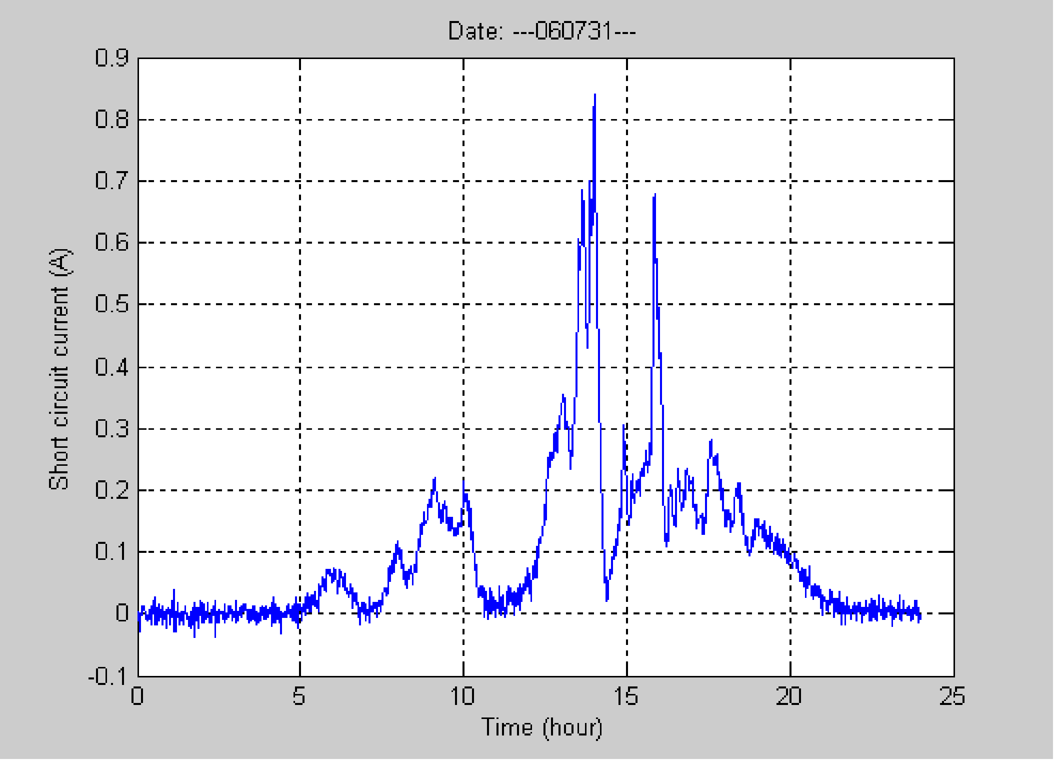

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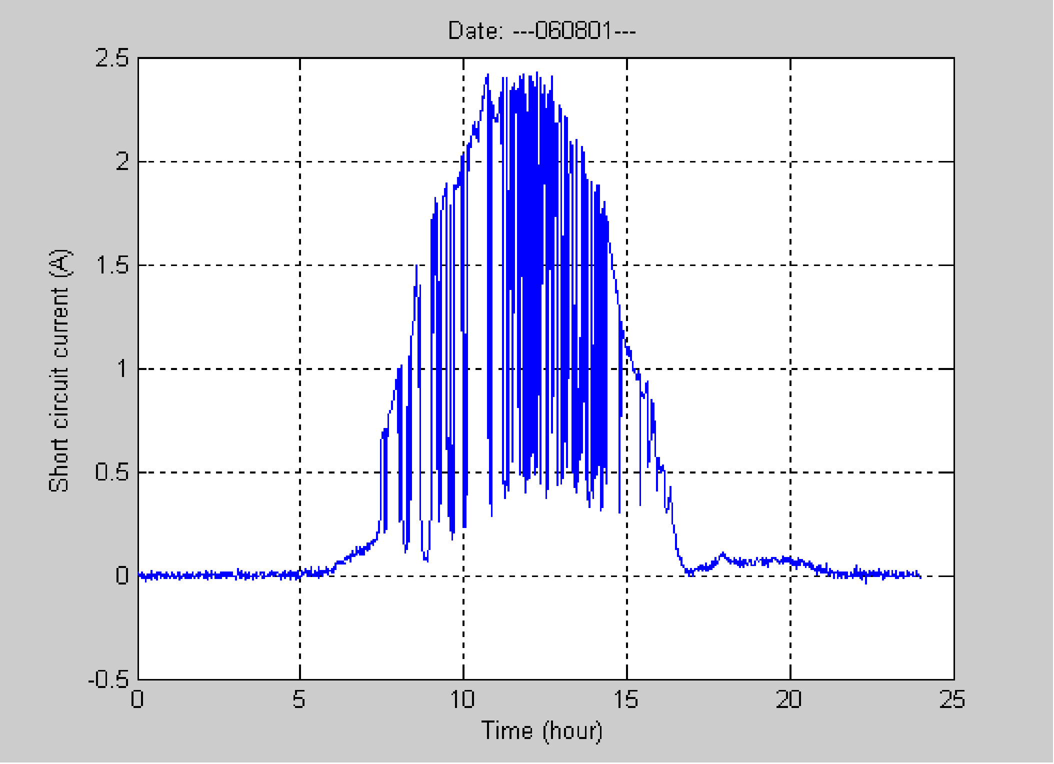
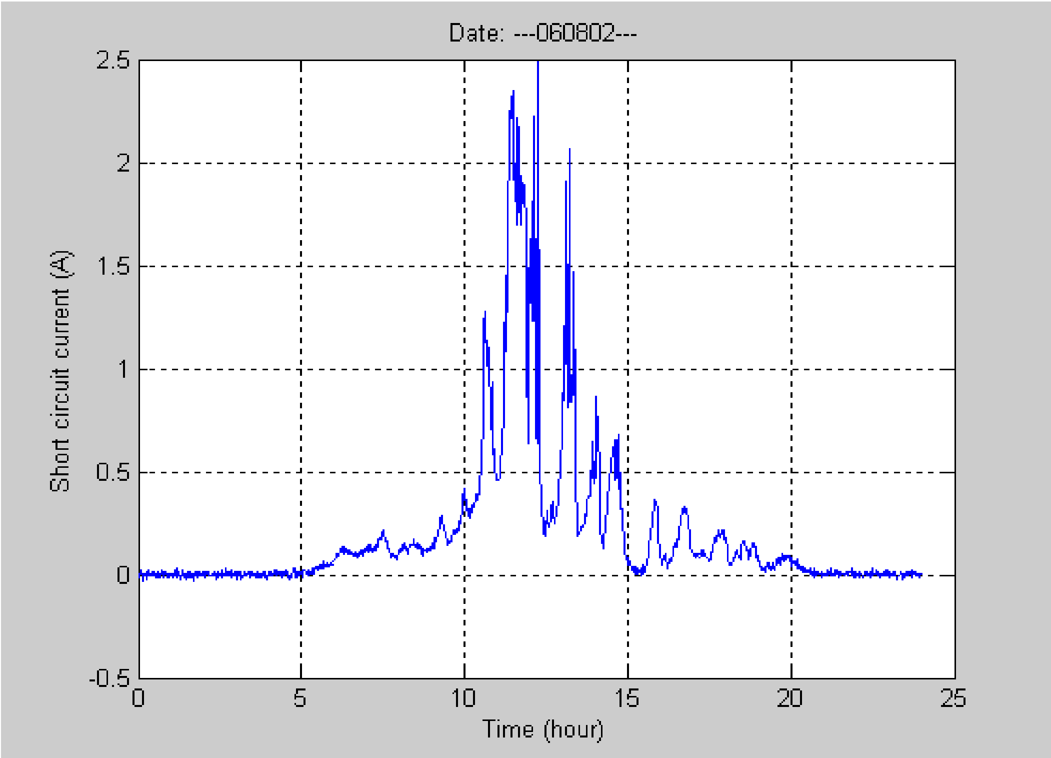


Figure 39 Short circuit current 060801



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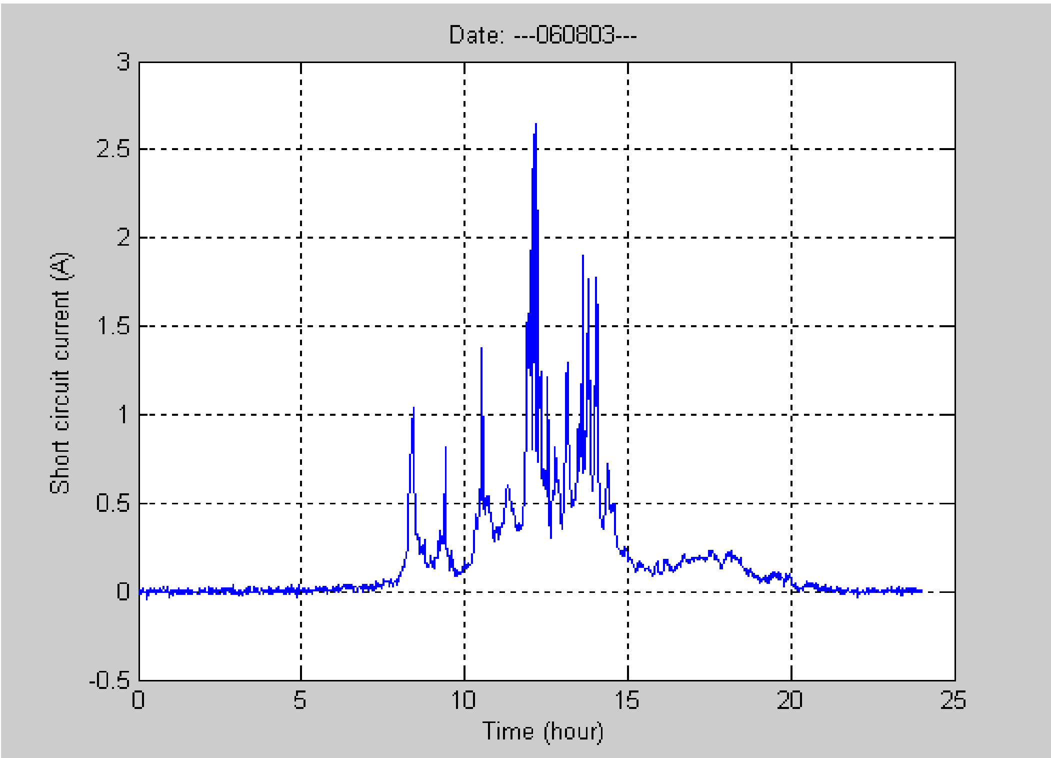

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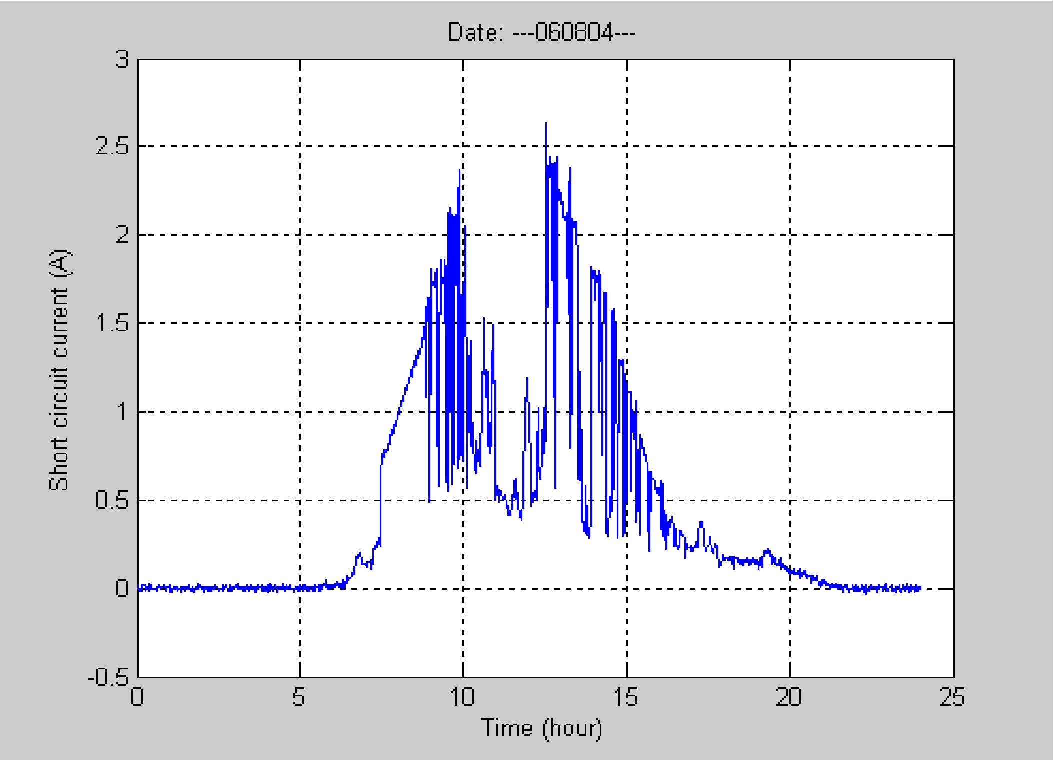

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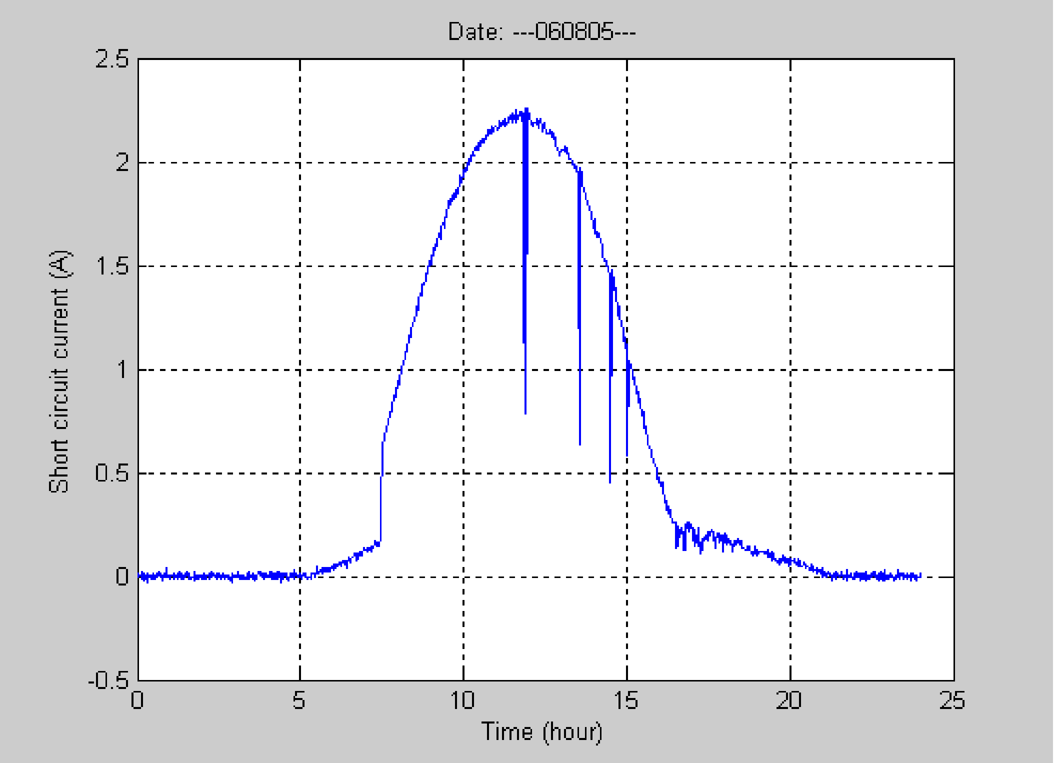

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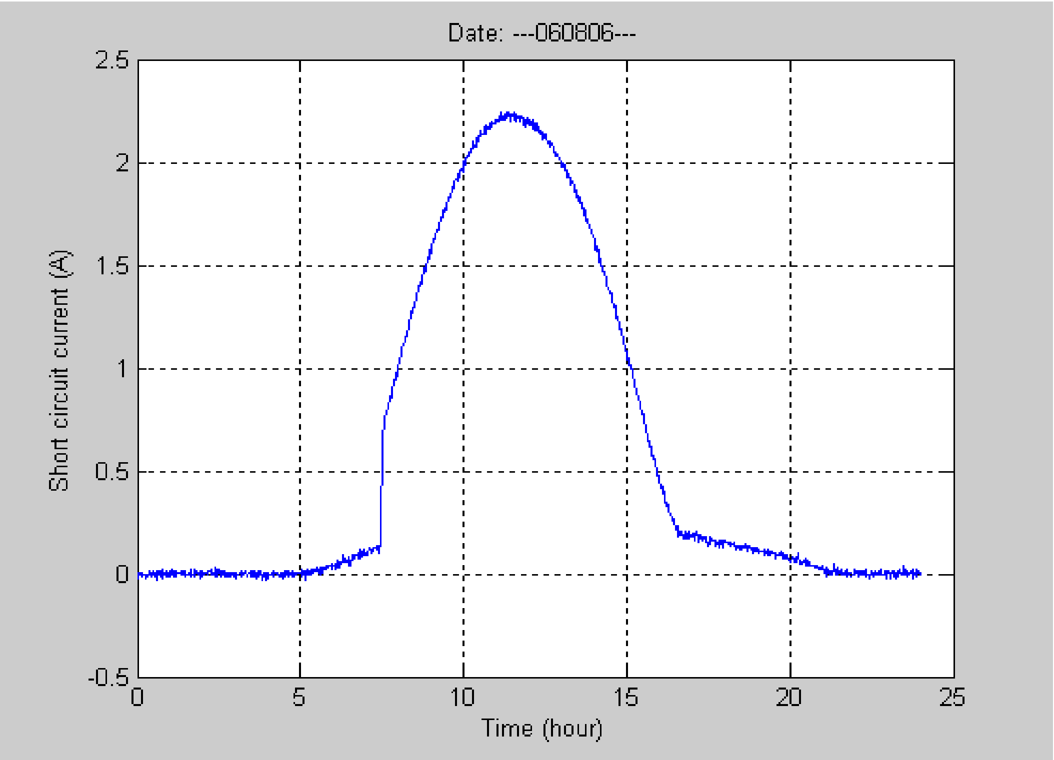

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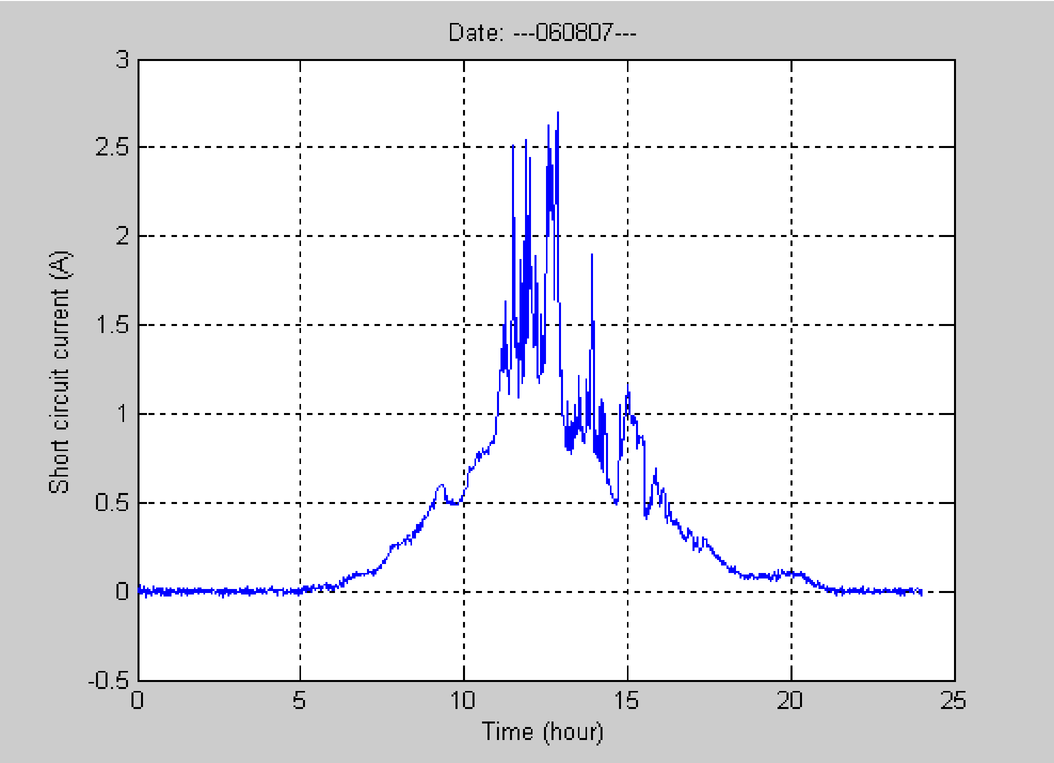

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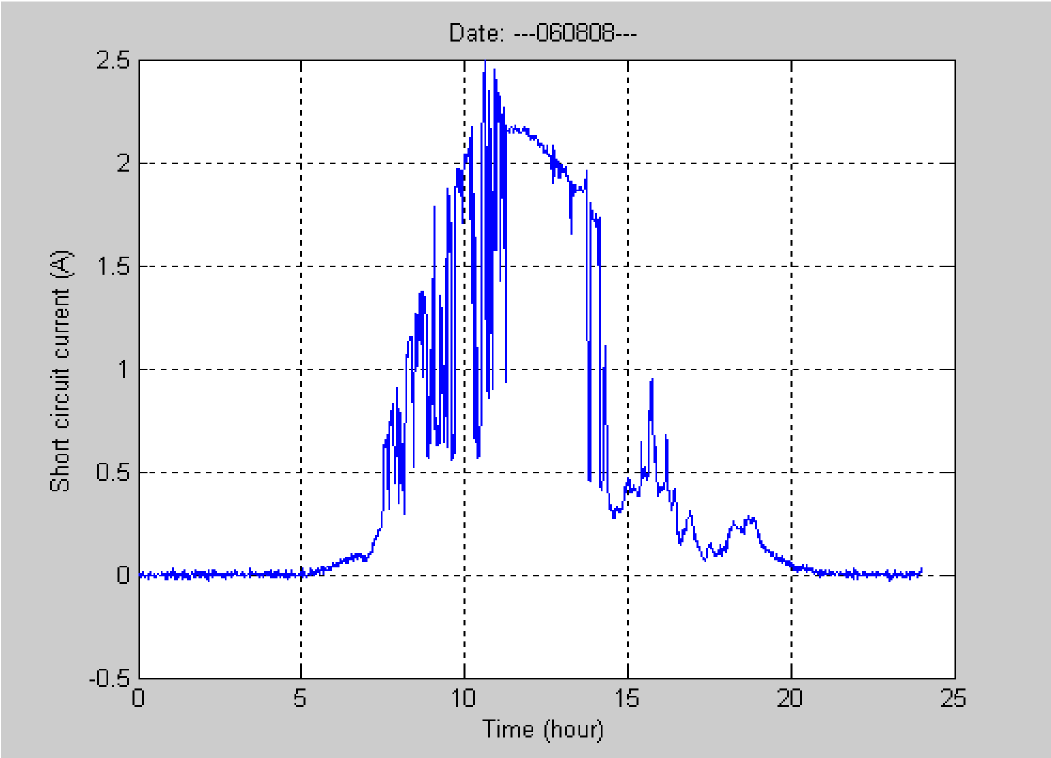

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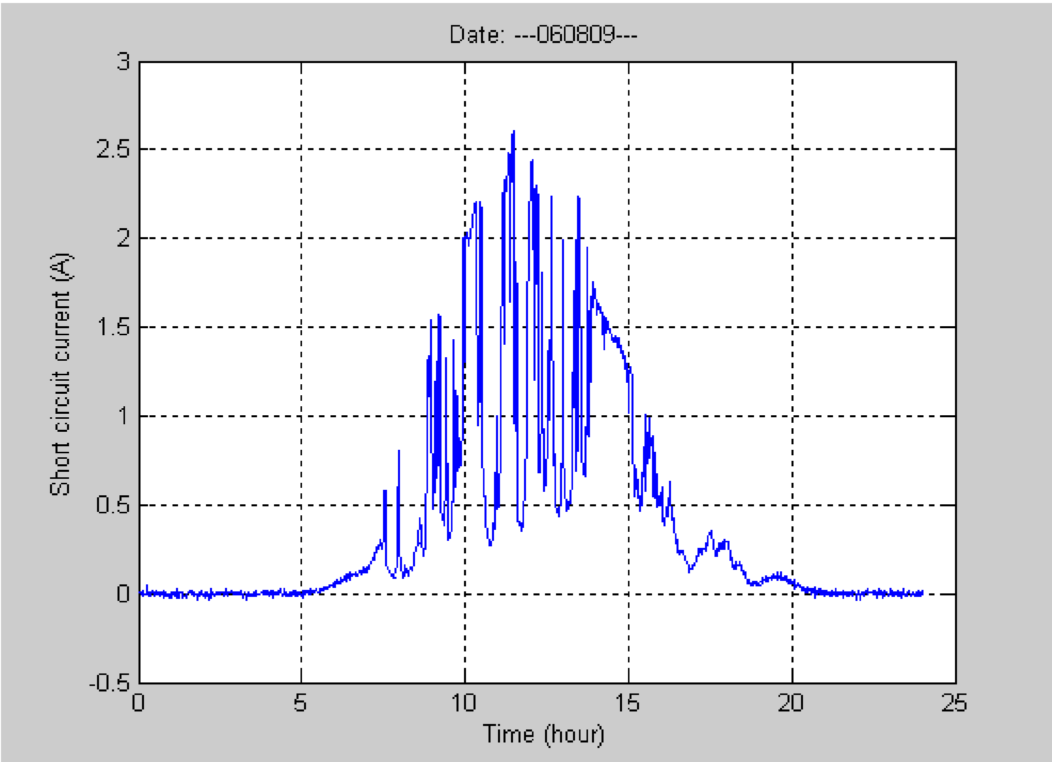

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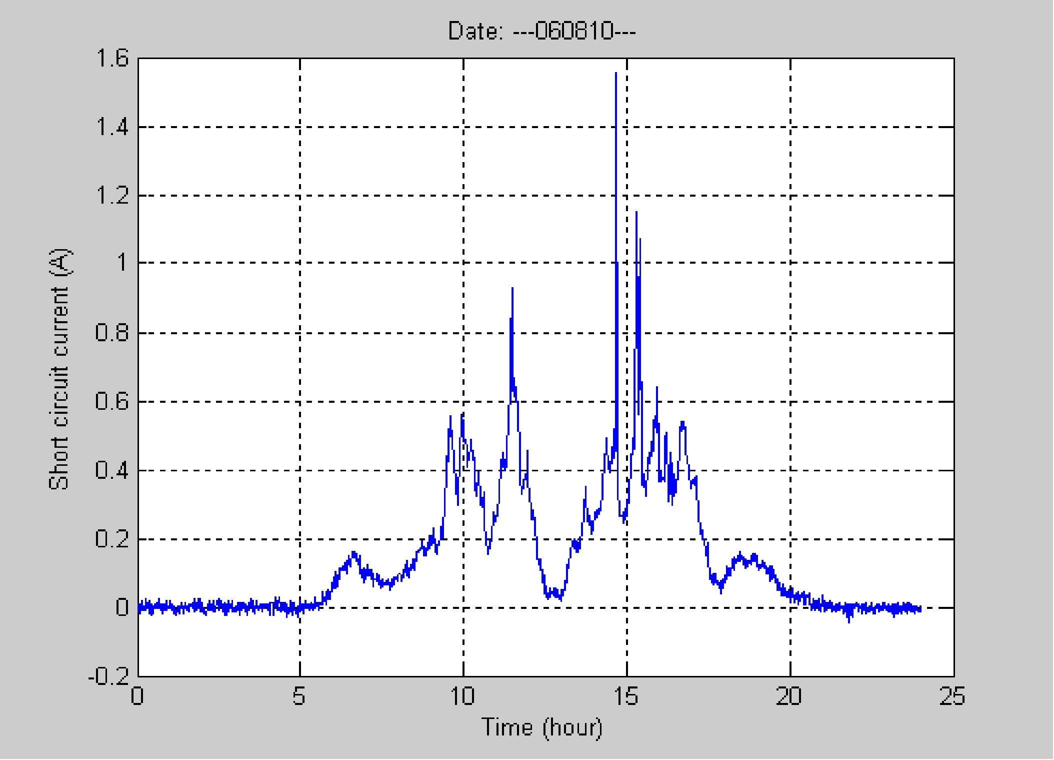

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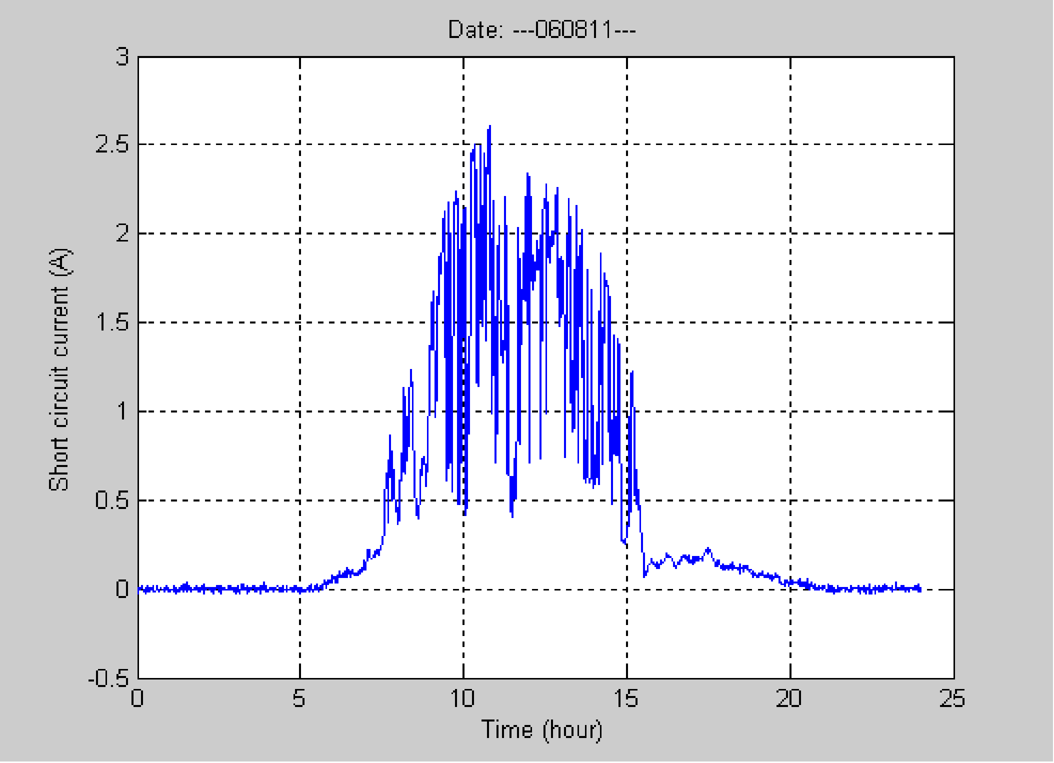

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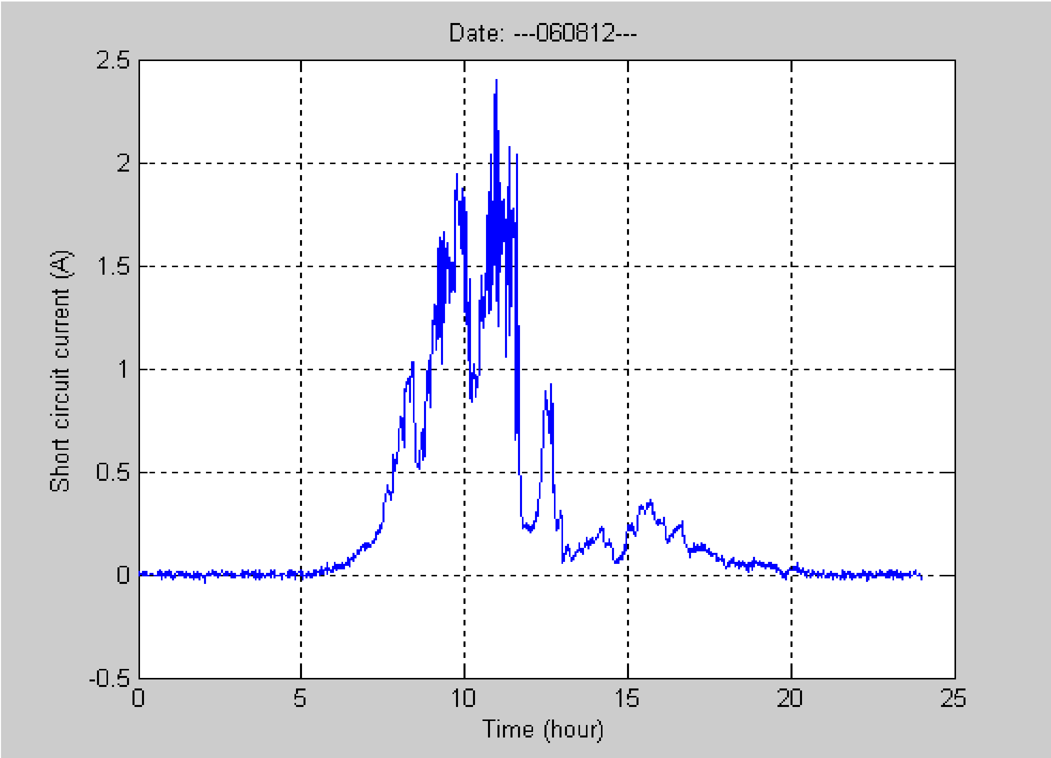

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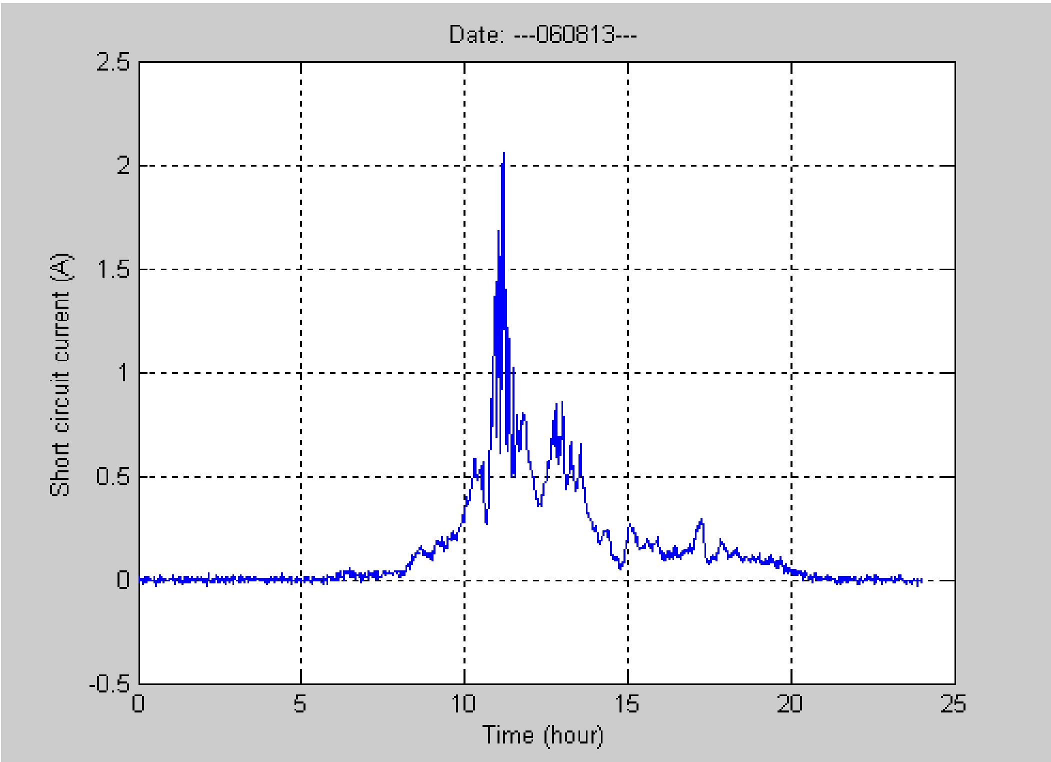

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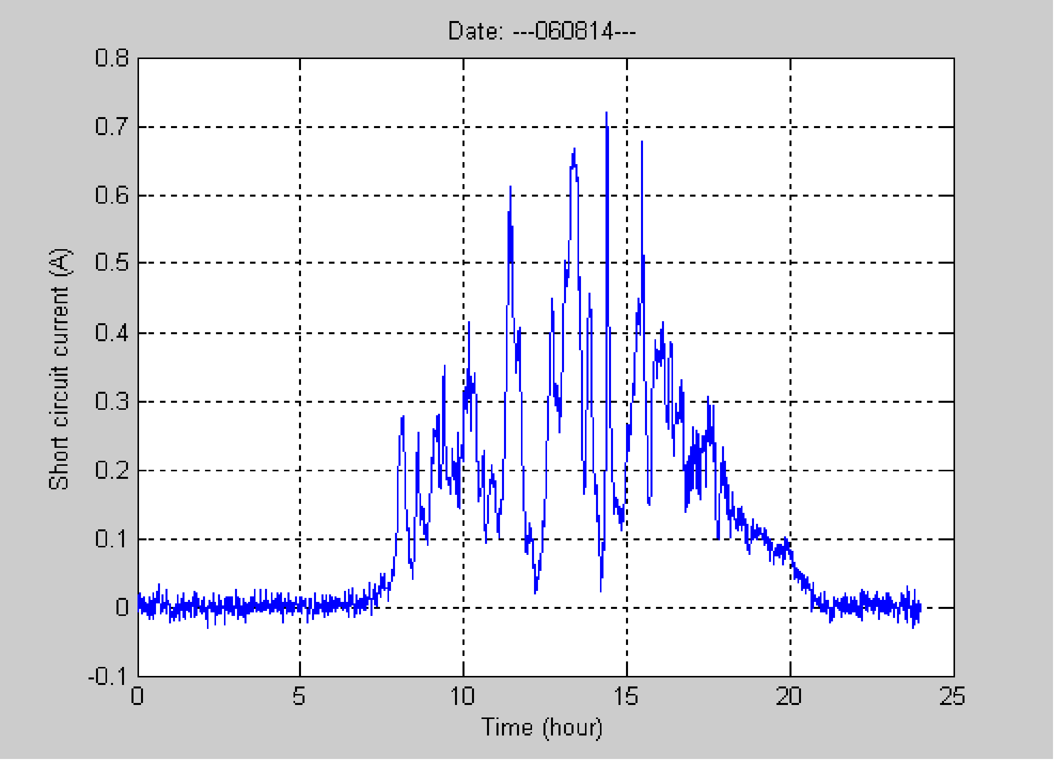

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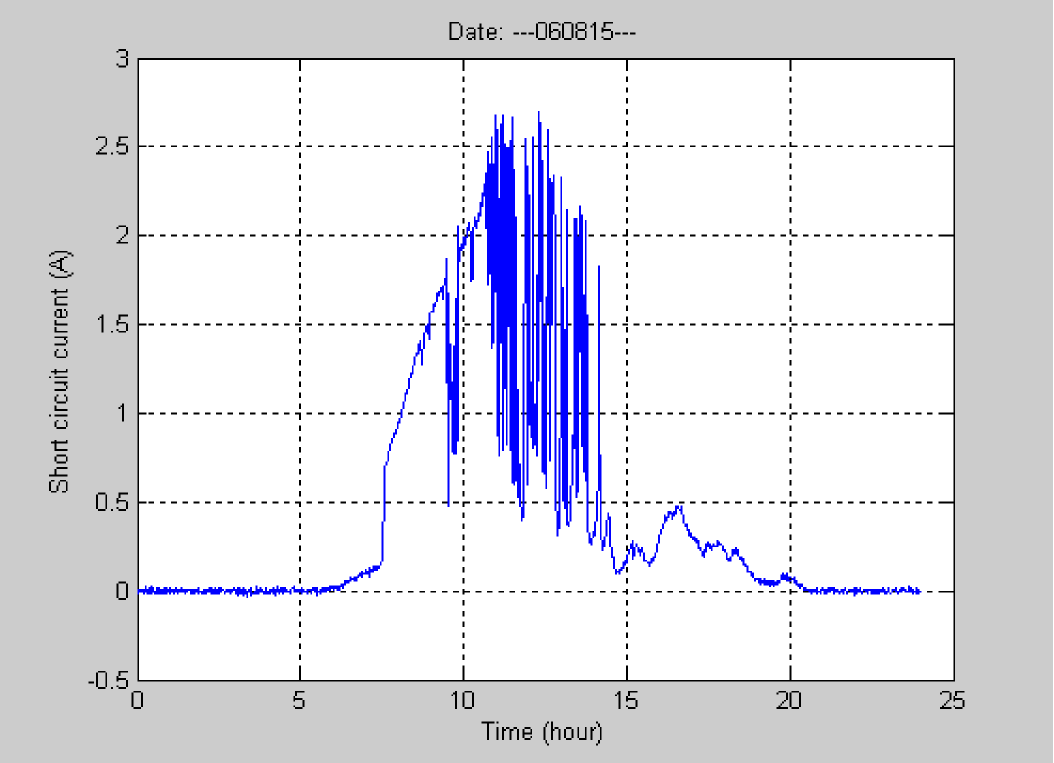

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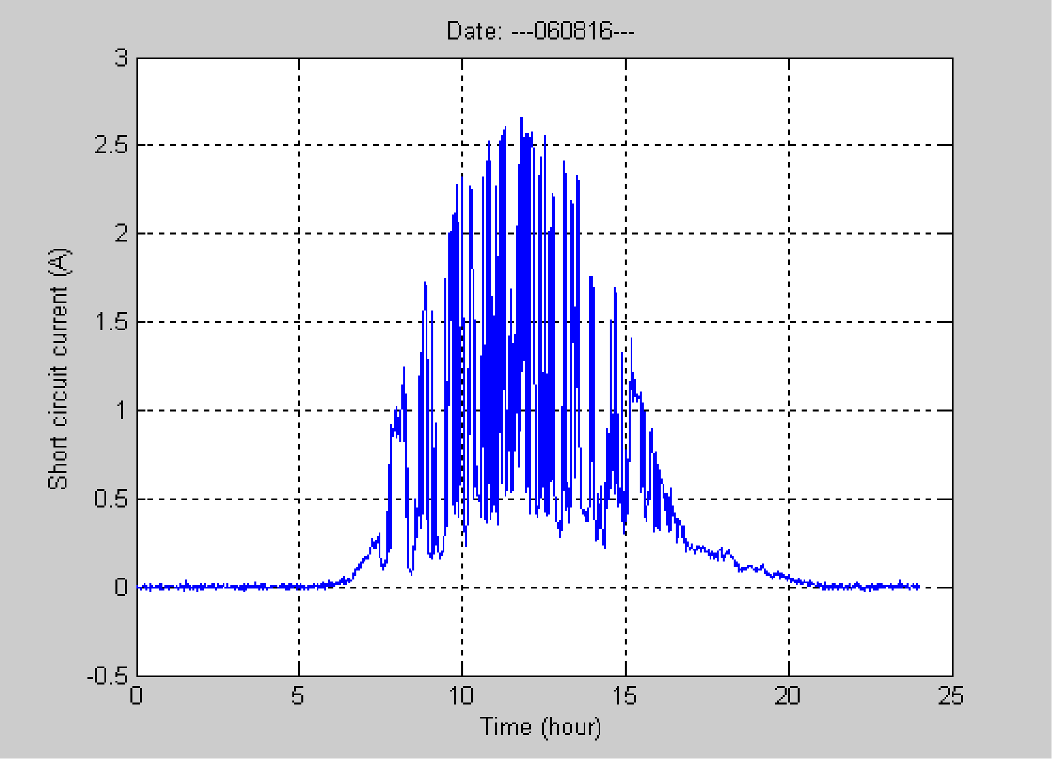

Figure 54 Short circuit current 060816

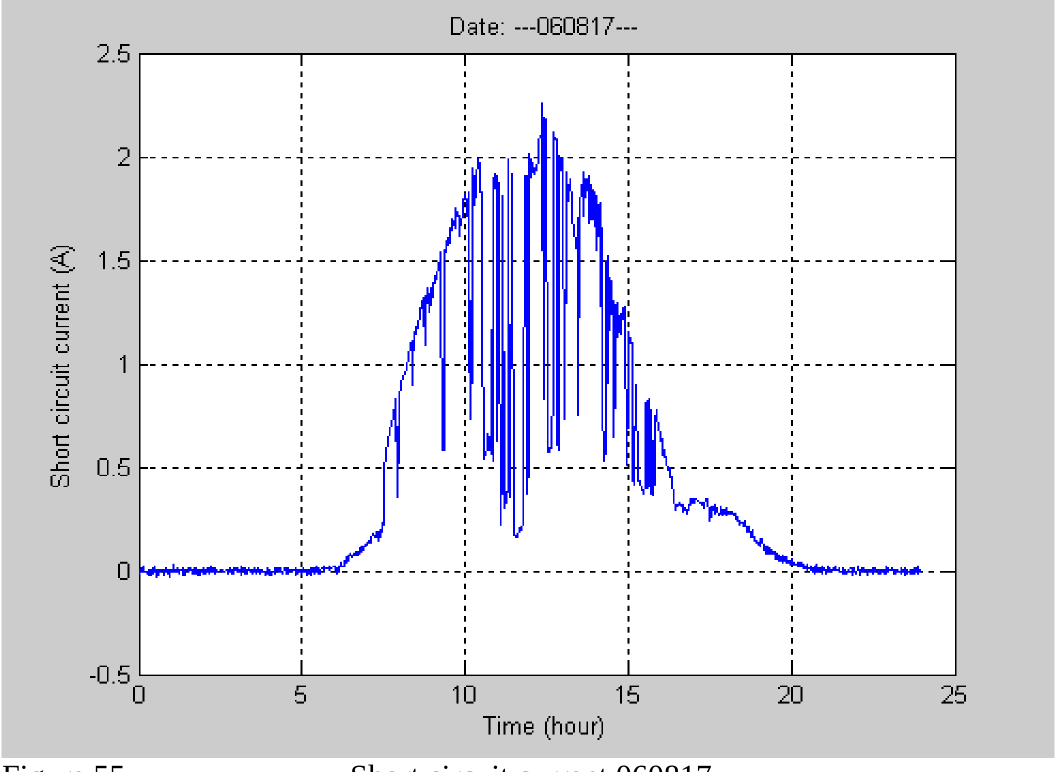

Figure 55 Short circuit current 060817

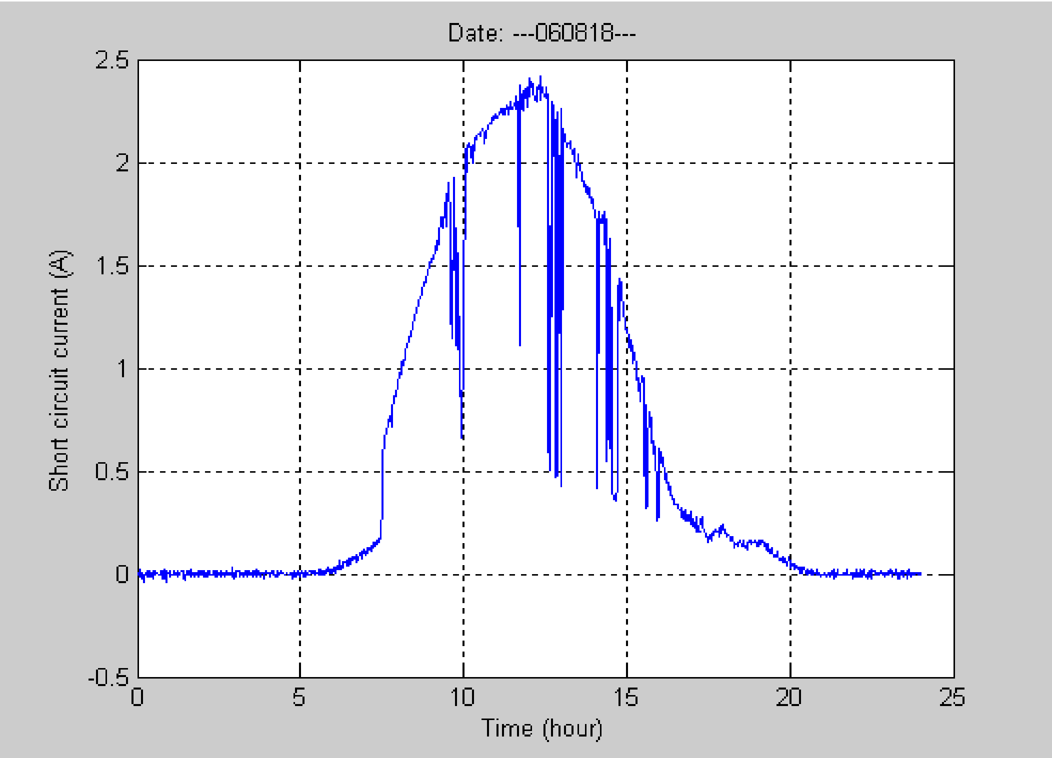

Figure 56 Short circuit current 060818

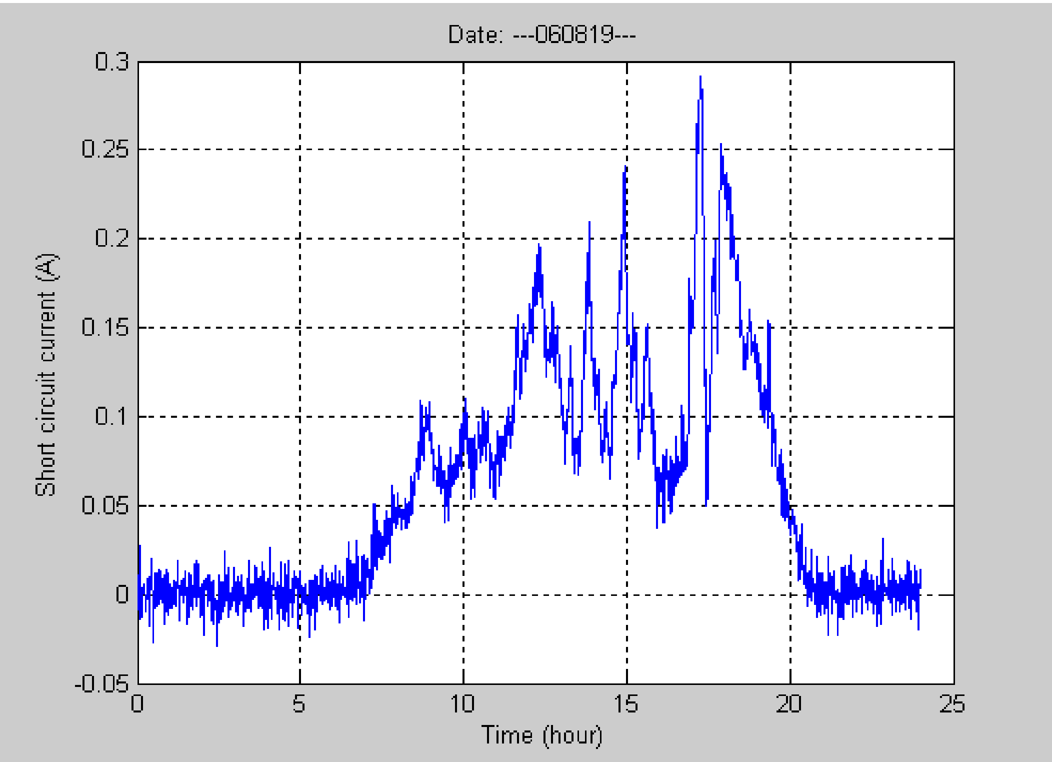

Figure 57 Short circuit current 060819

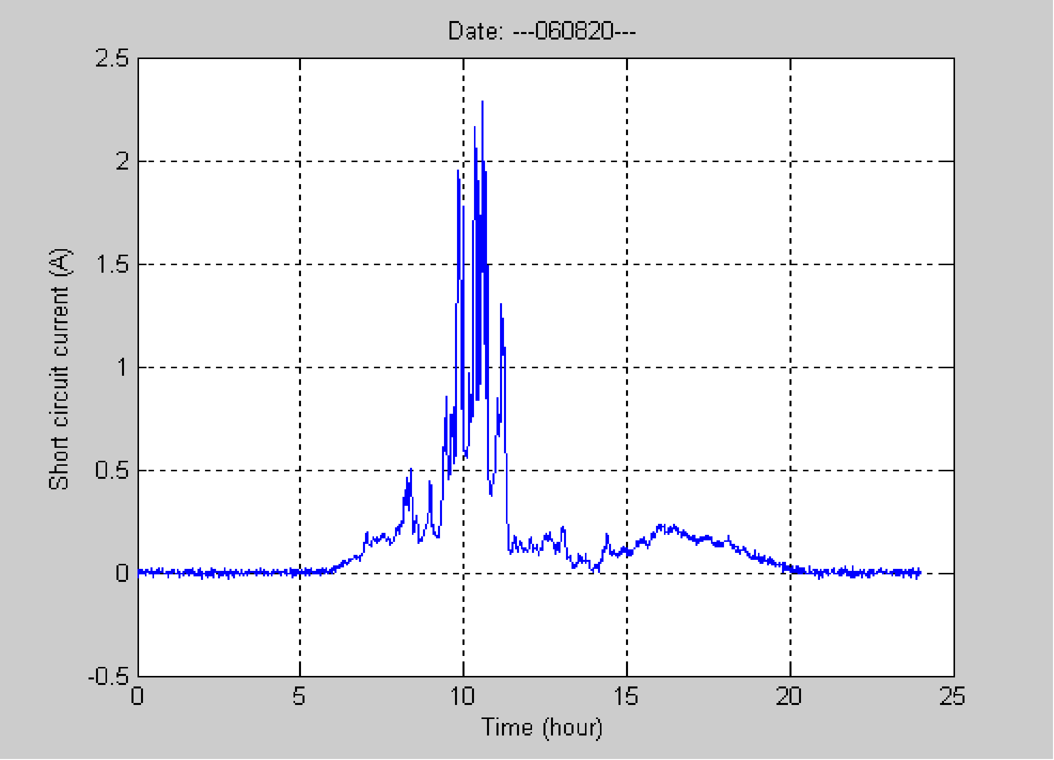

Figure 58 Short circuit current 060820

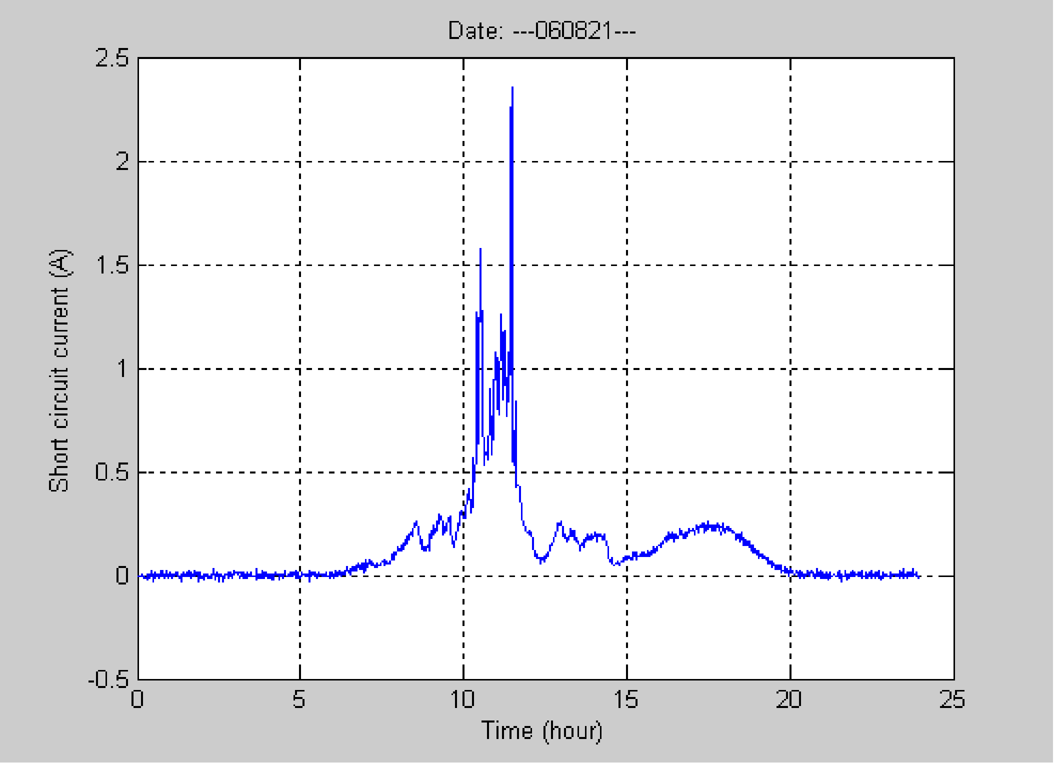

Figure 59 Short circuit current 060821

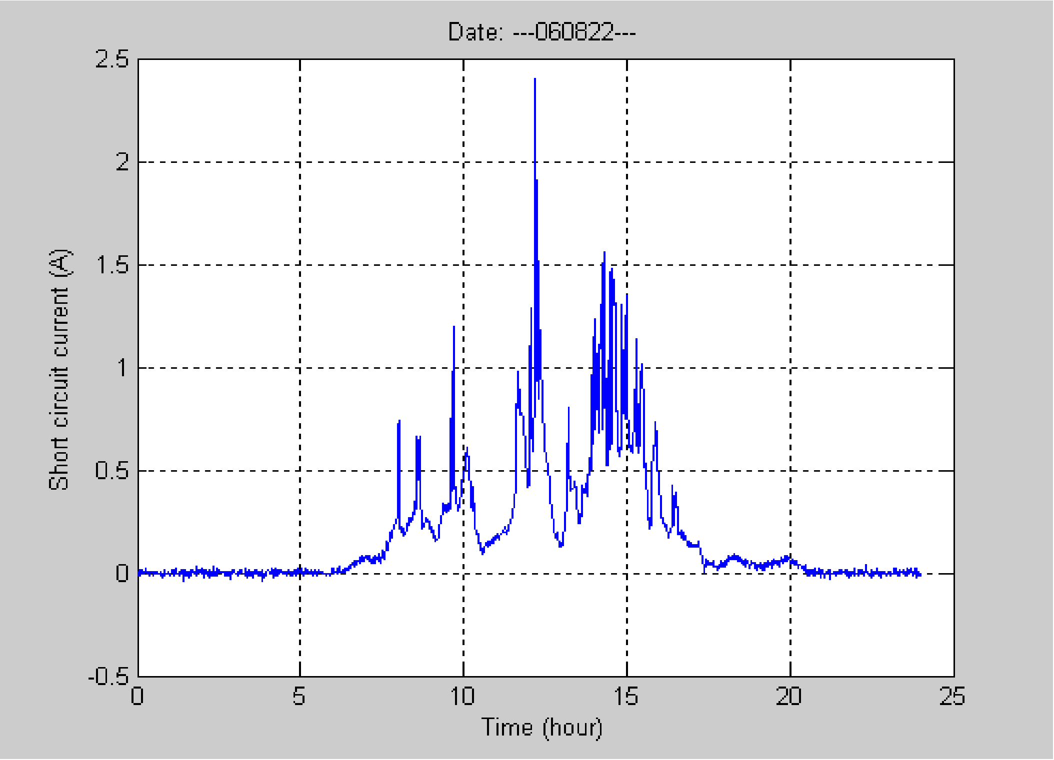

Figure 60 Short circuit current 060822

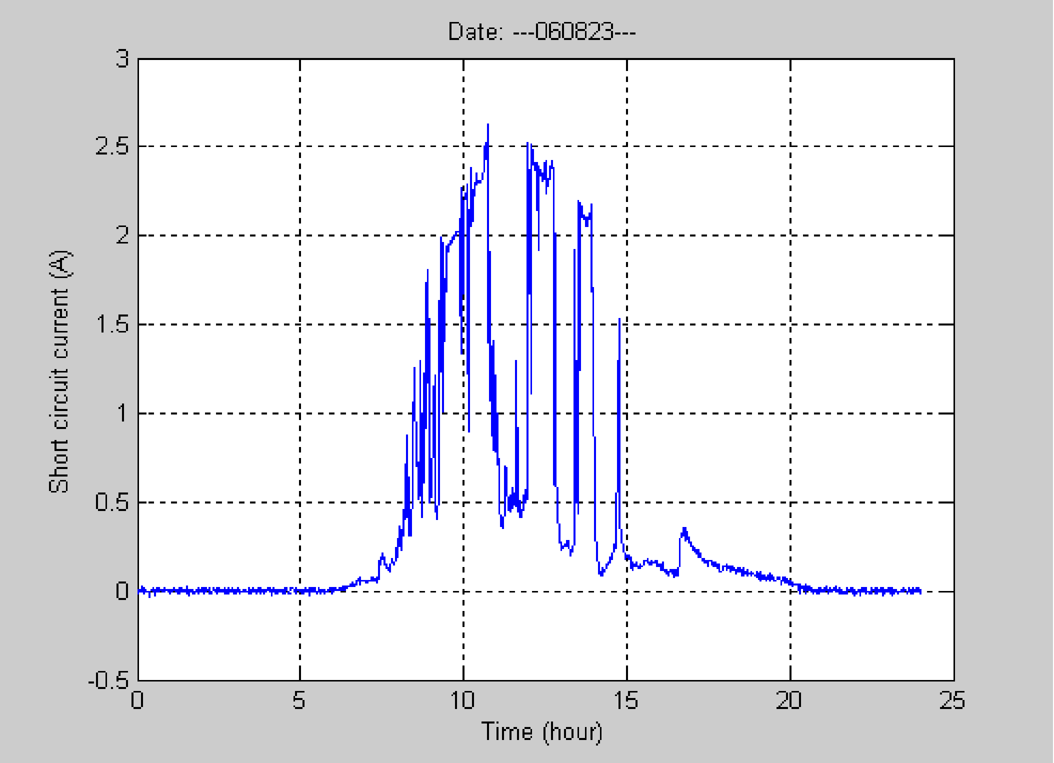

Figure 61 Short circuit current 060823

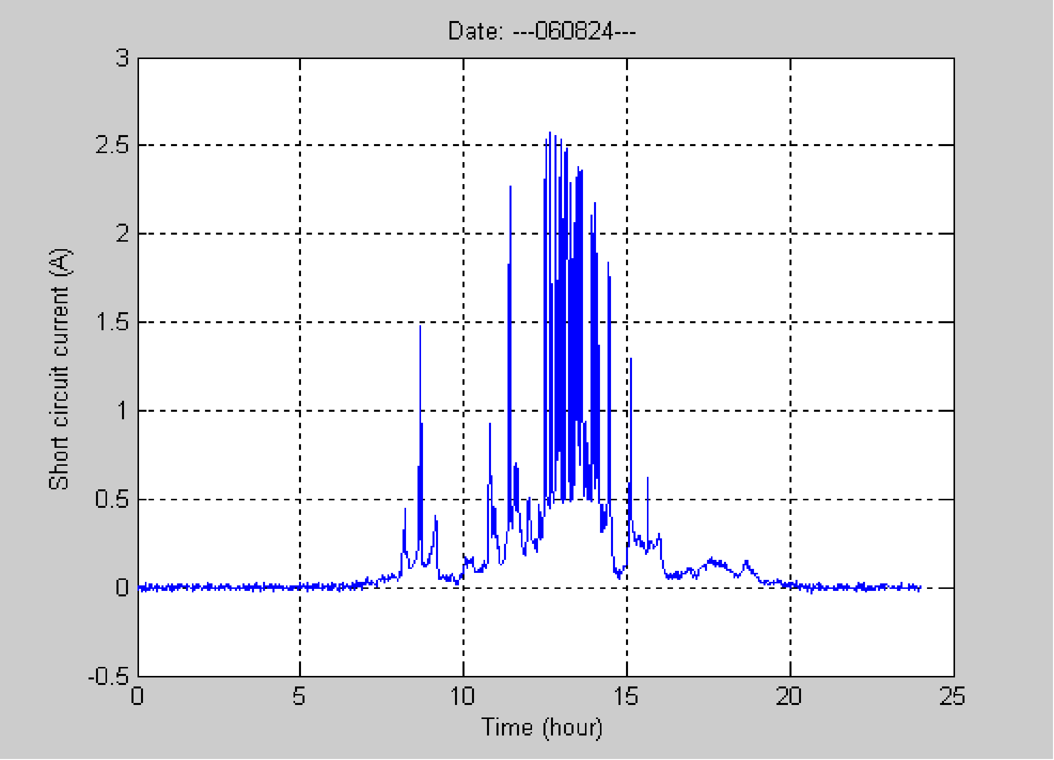

Figure 62 Short circuit current 060824

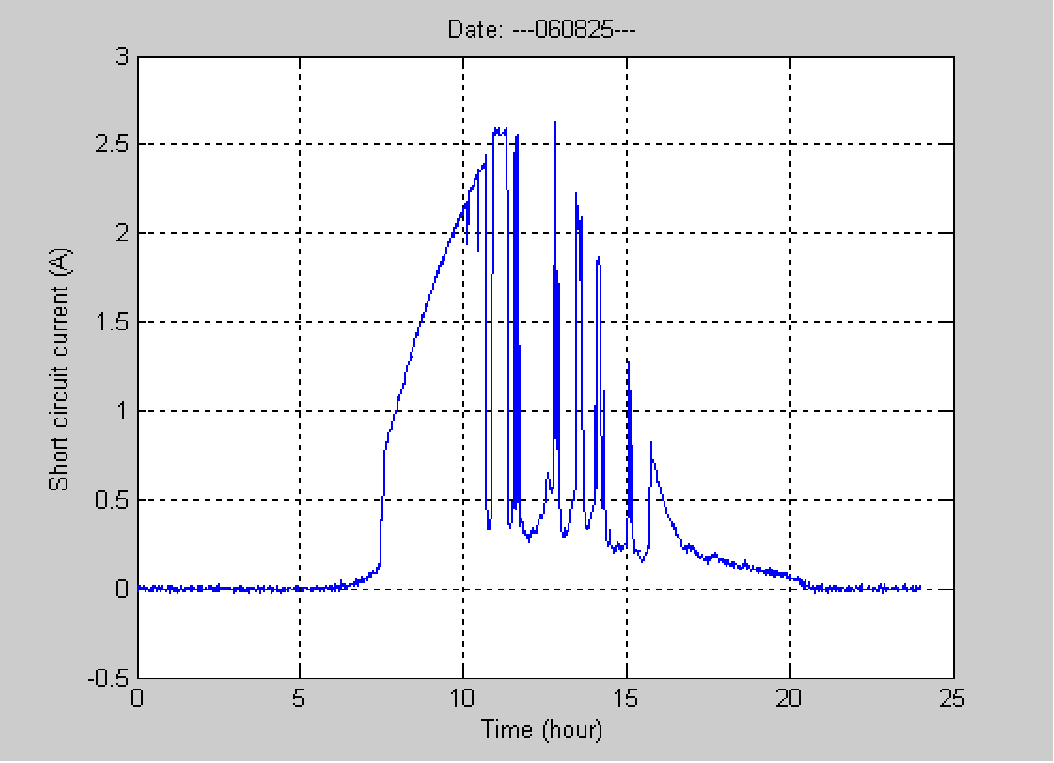

Figure 63 Short circuit current 060825

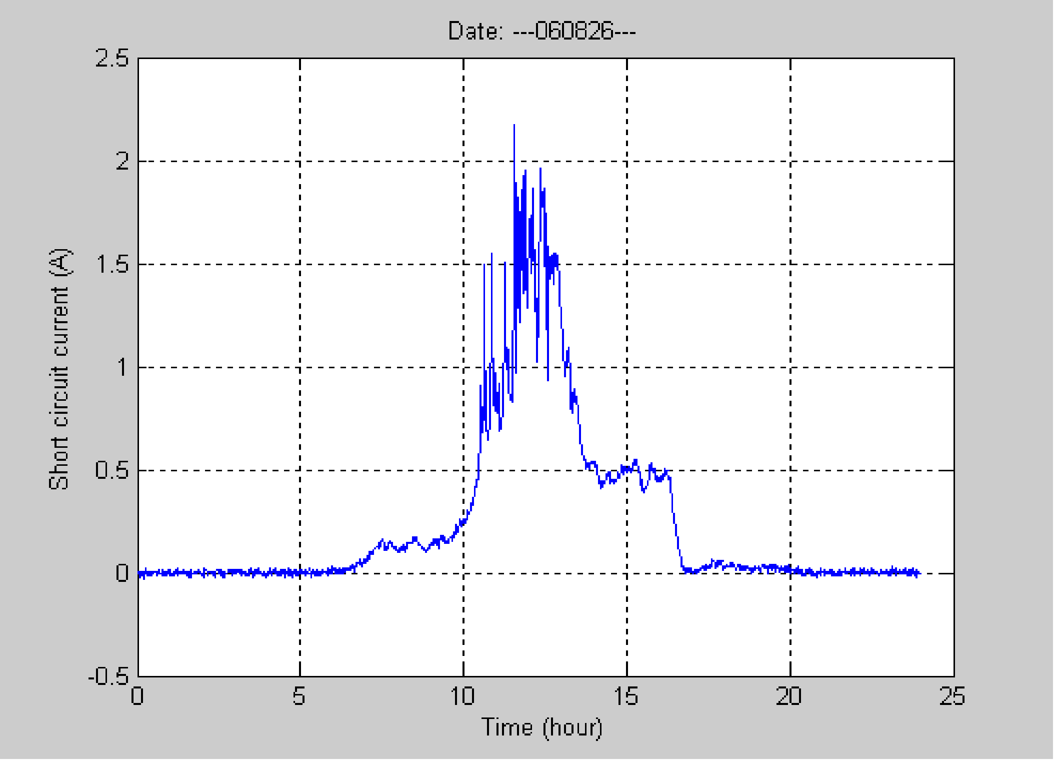

Figure 64 Short circuit current 060826

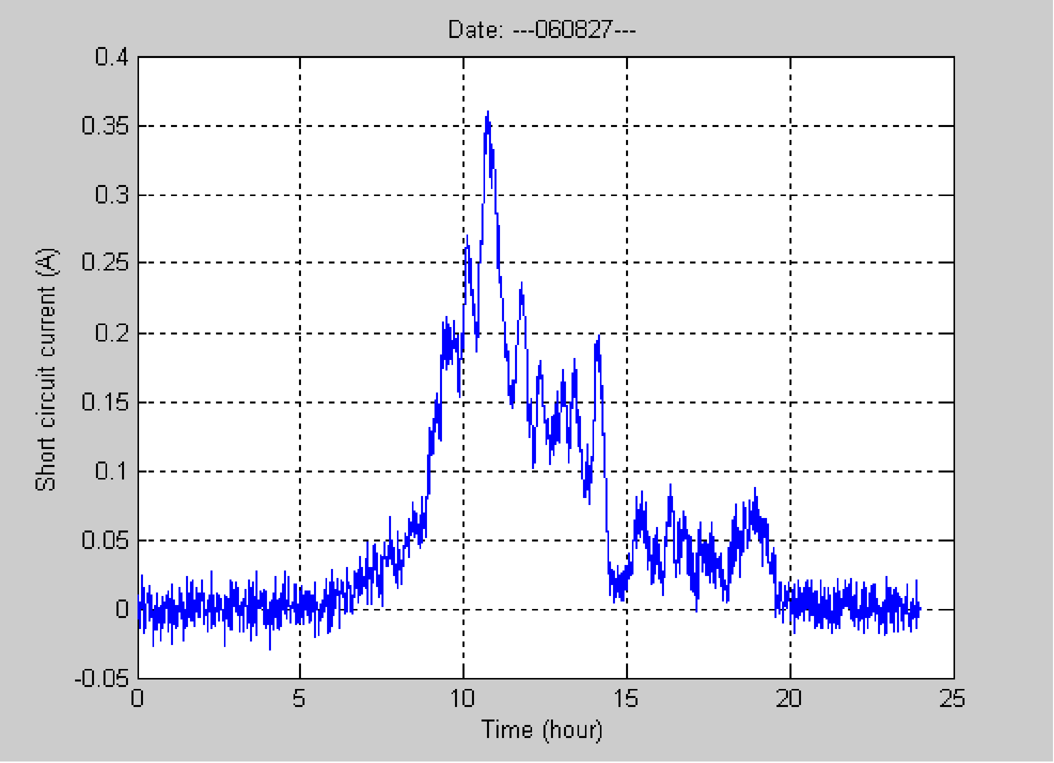

Figure 65 Short circuit current 060827

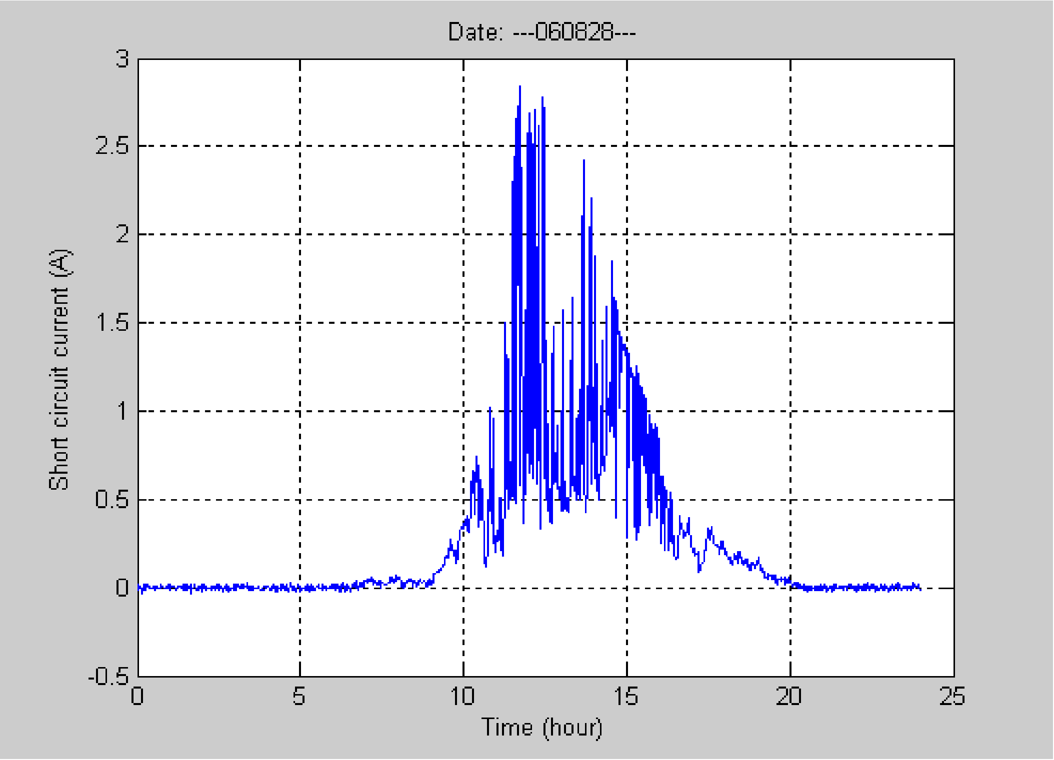

Figure 66 Short circuit current 060828

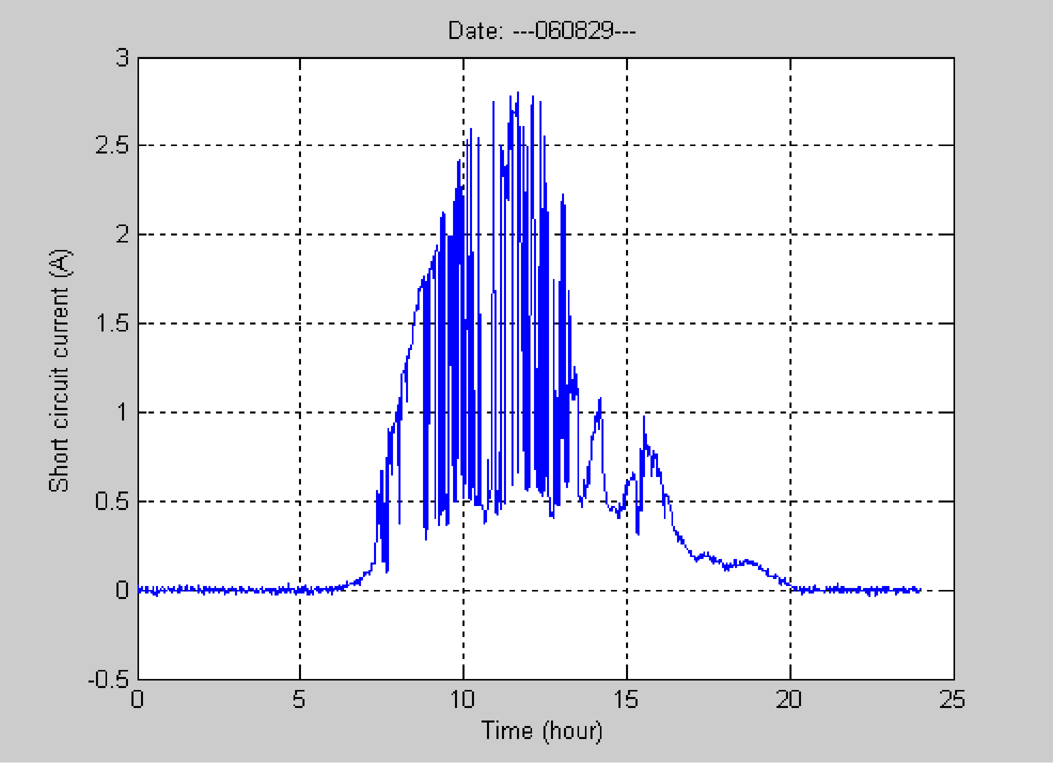

Figure 67 Short circuit current 060829

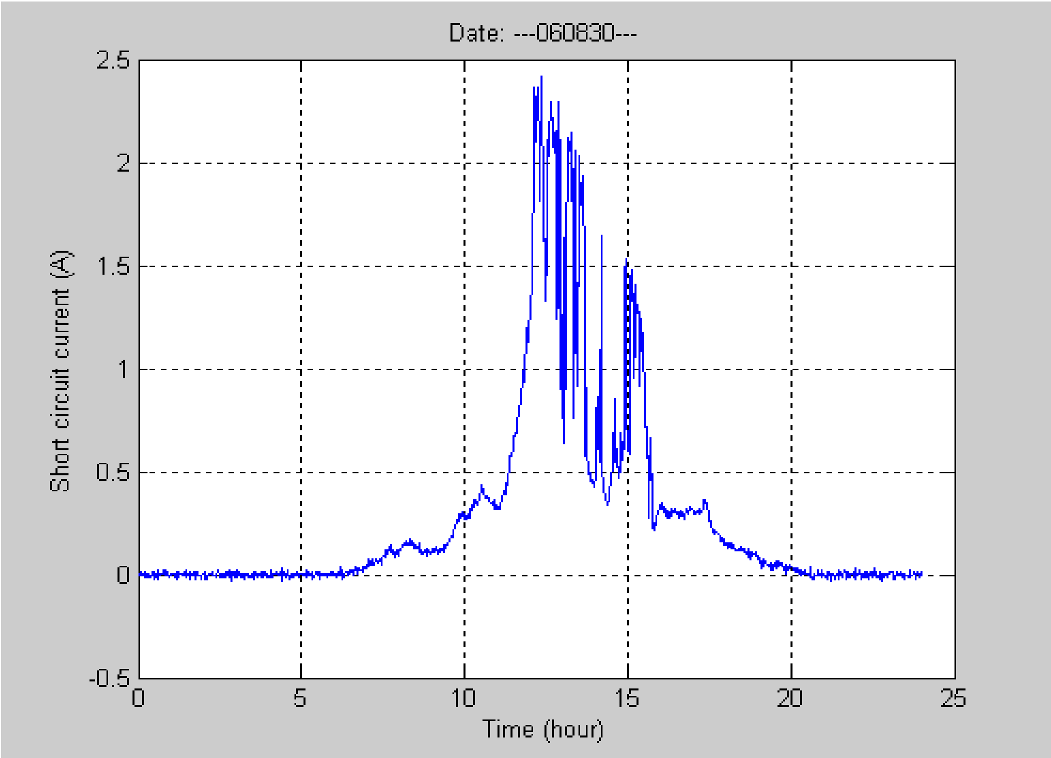

Figure 68 Short circuit current 060830

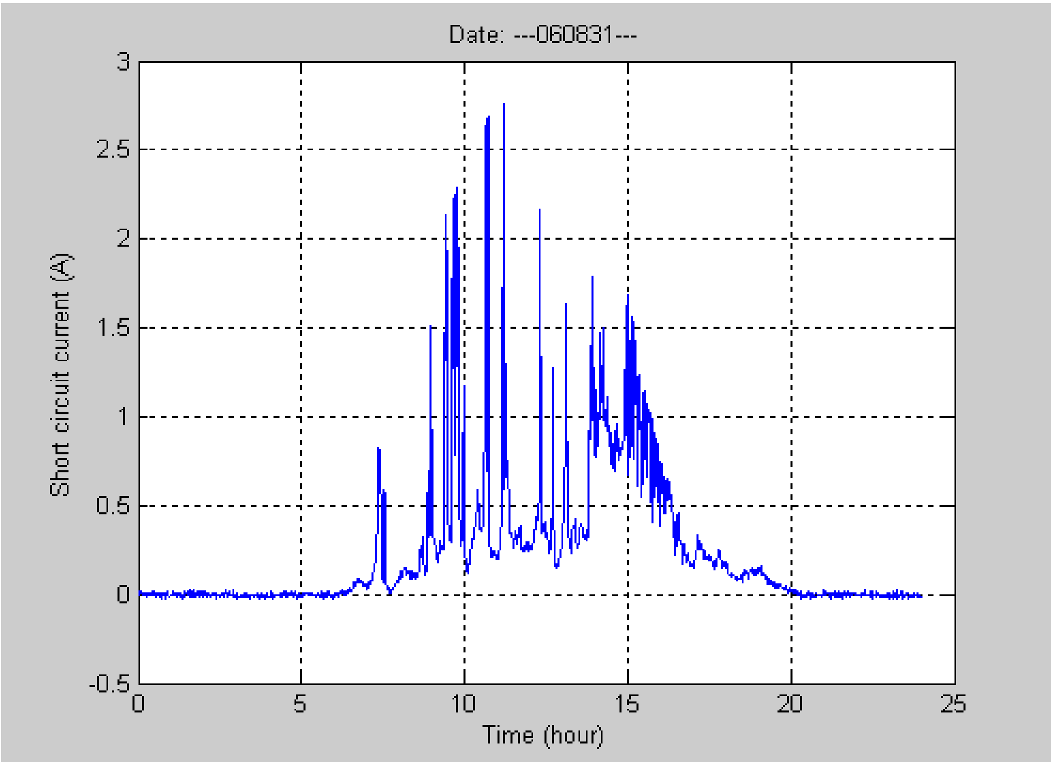

Figure 69 Short circuit current 060831

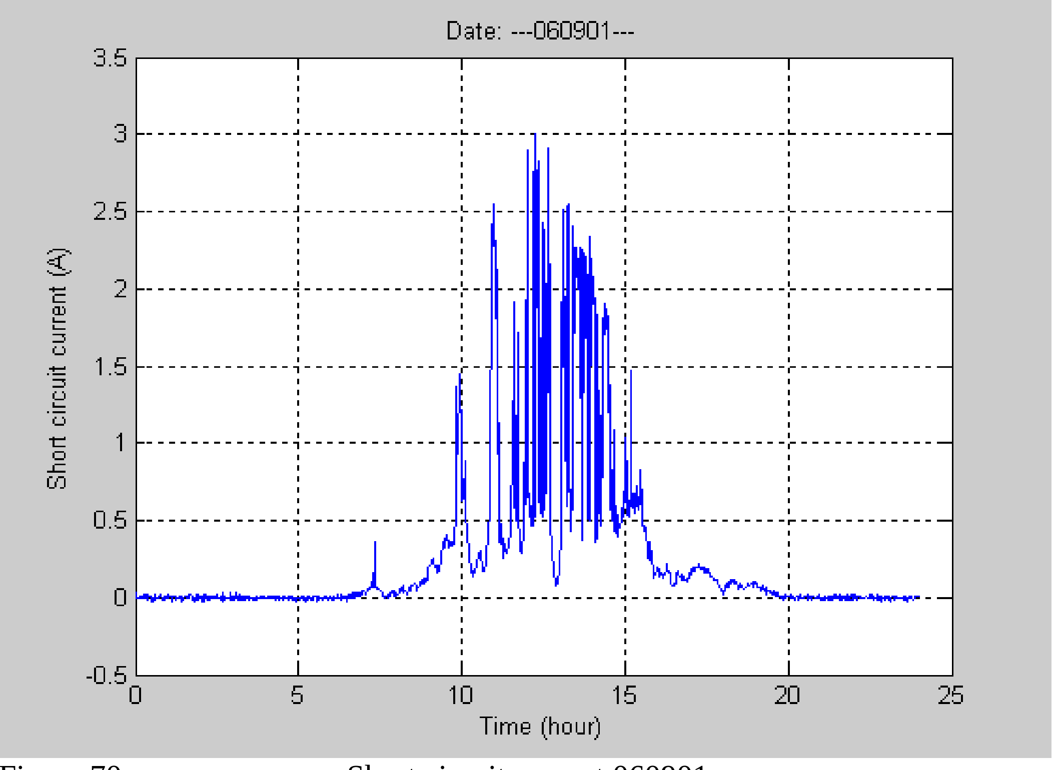

Figure 70 Short circuit current 060901

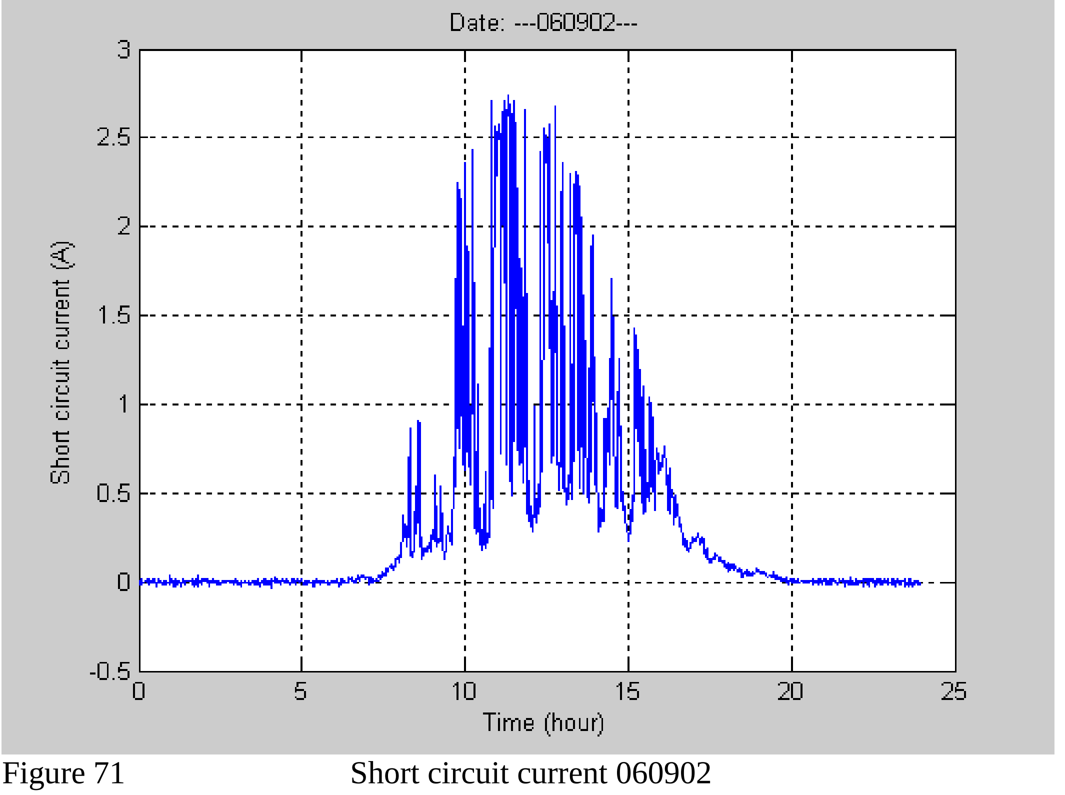

Figure 71 Short circuit current 060902

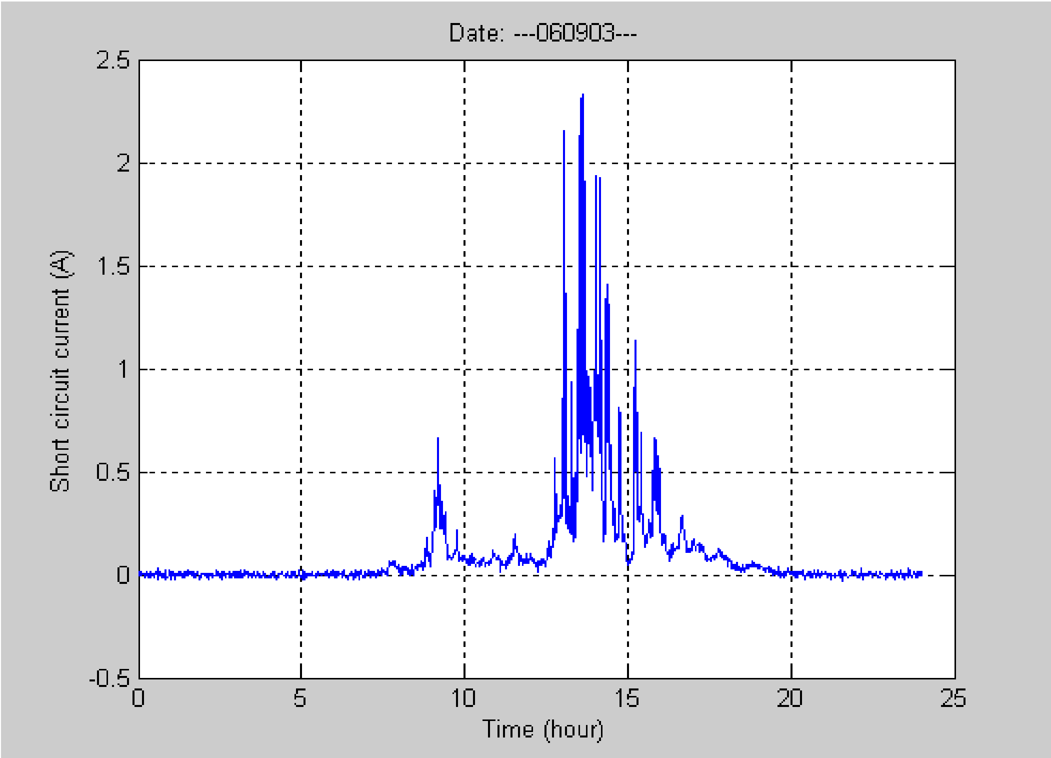

Figure 72 Short circuit current 060903

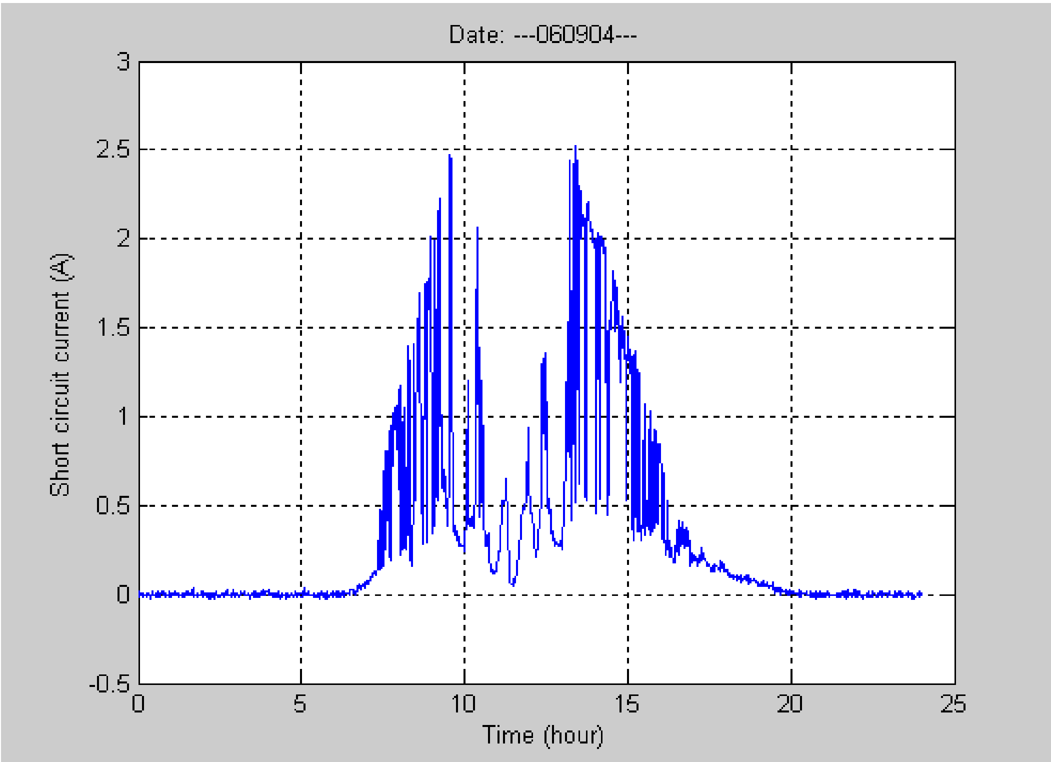

Figure 73 Short circuit current 060904

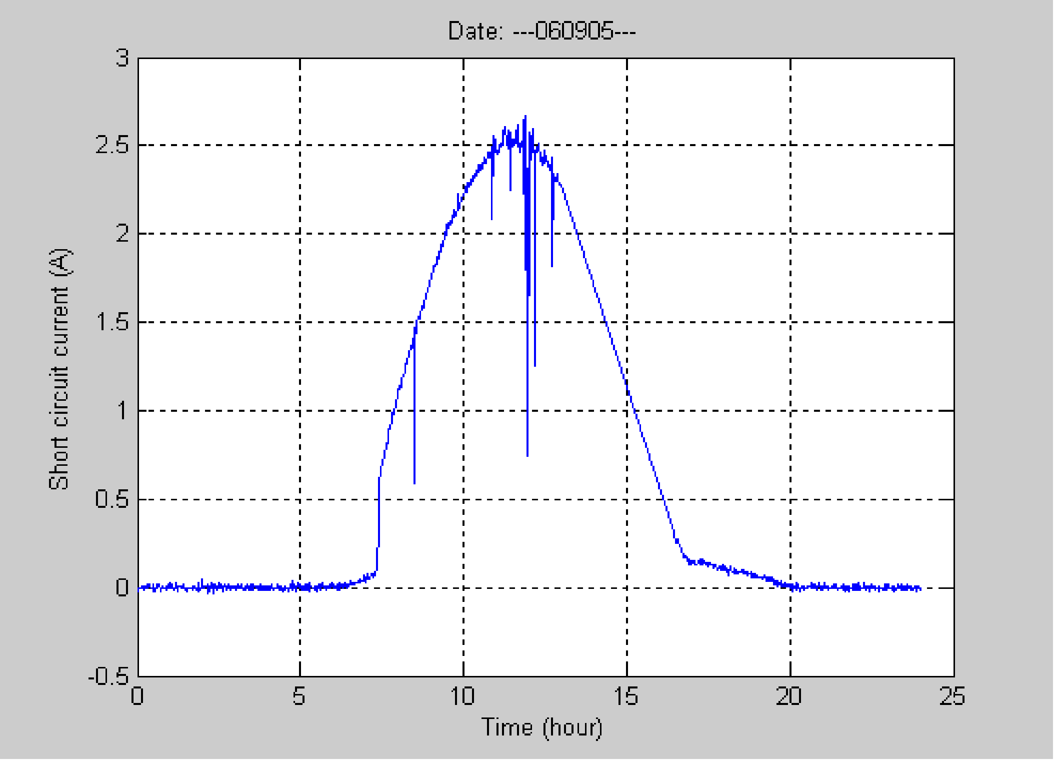

Figure 74 Short circuit current 060905

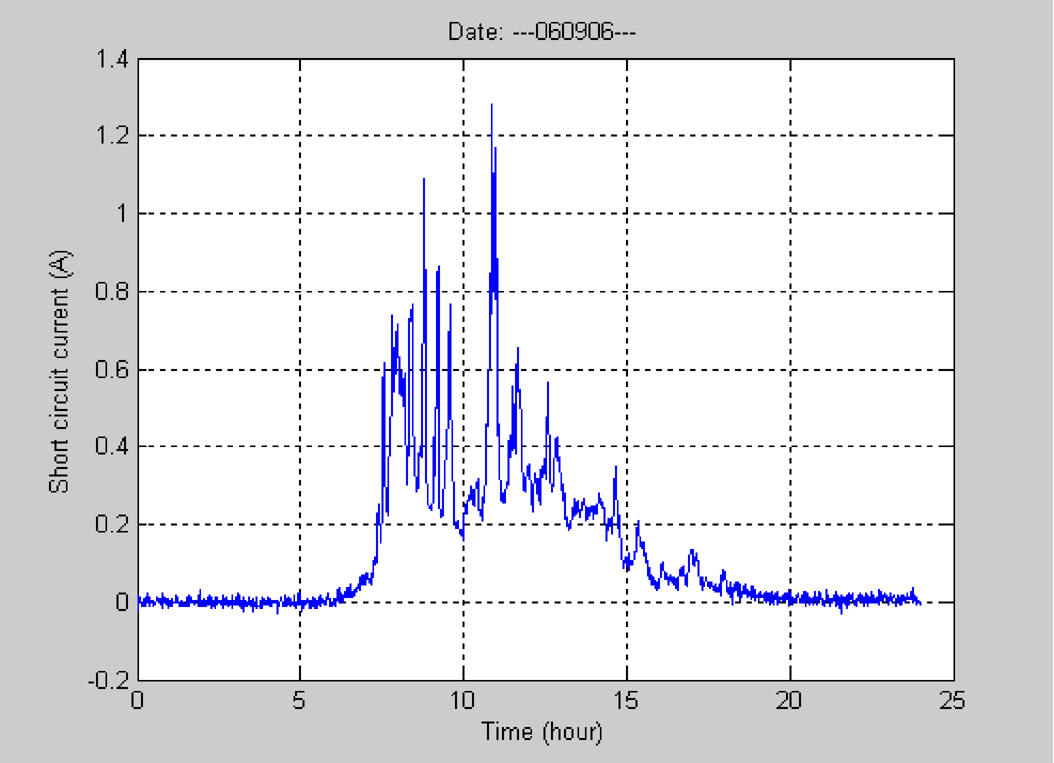

Figure 75 Short circuit current 060906

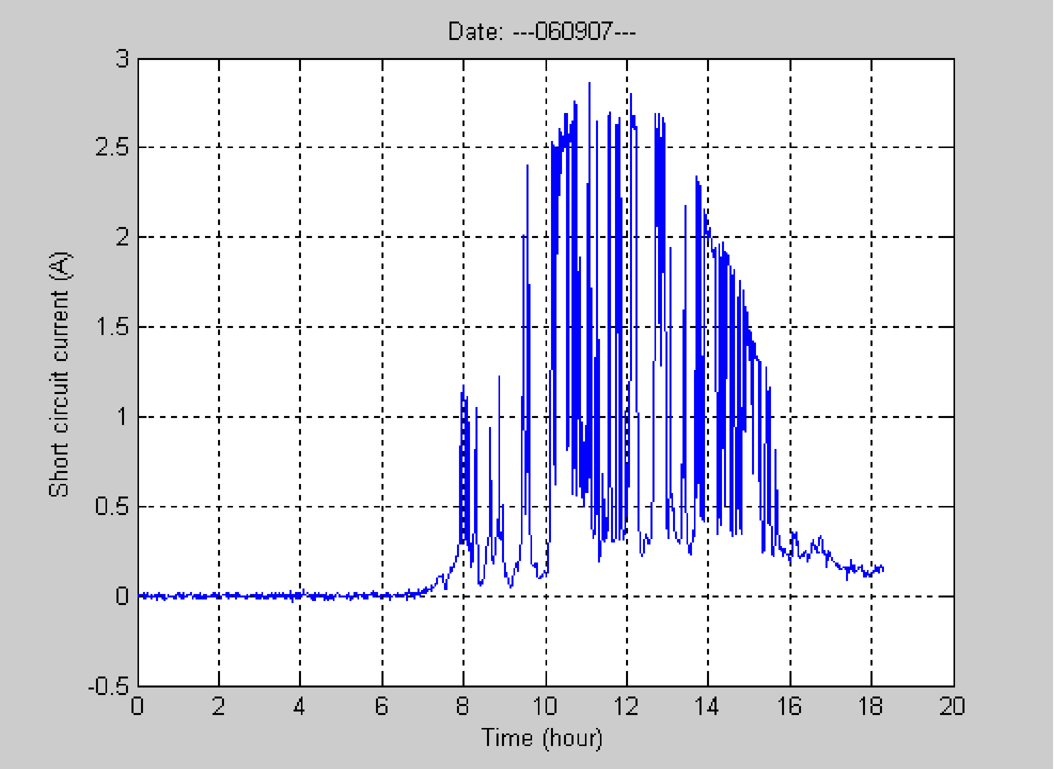

Figure 76 Short circuit current 060907

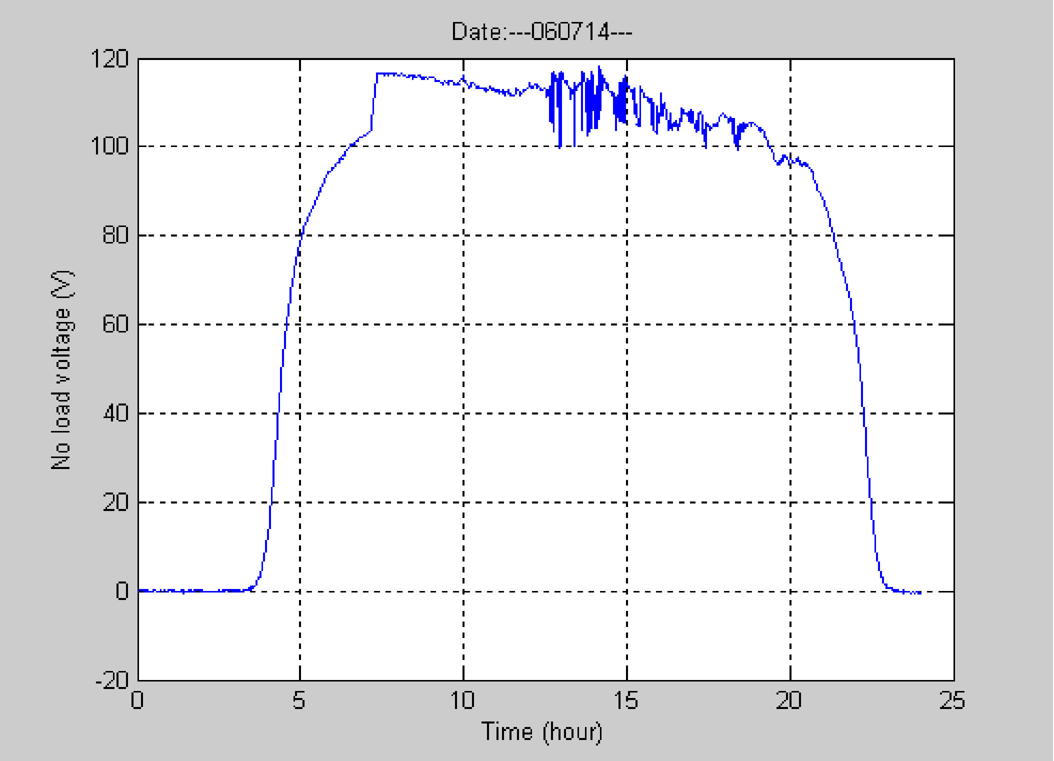

Figure 77 No load voltage 060714

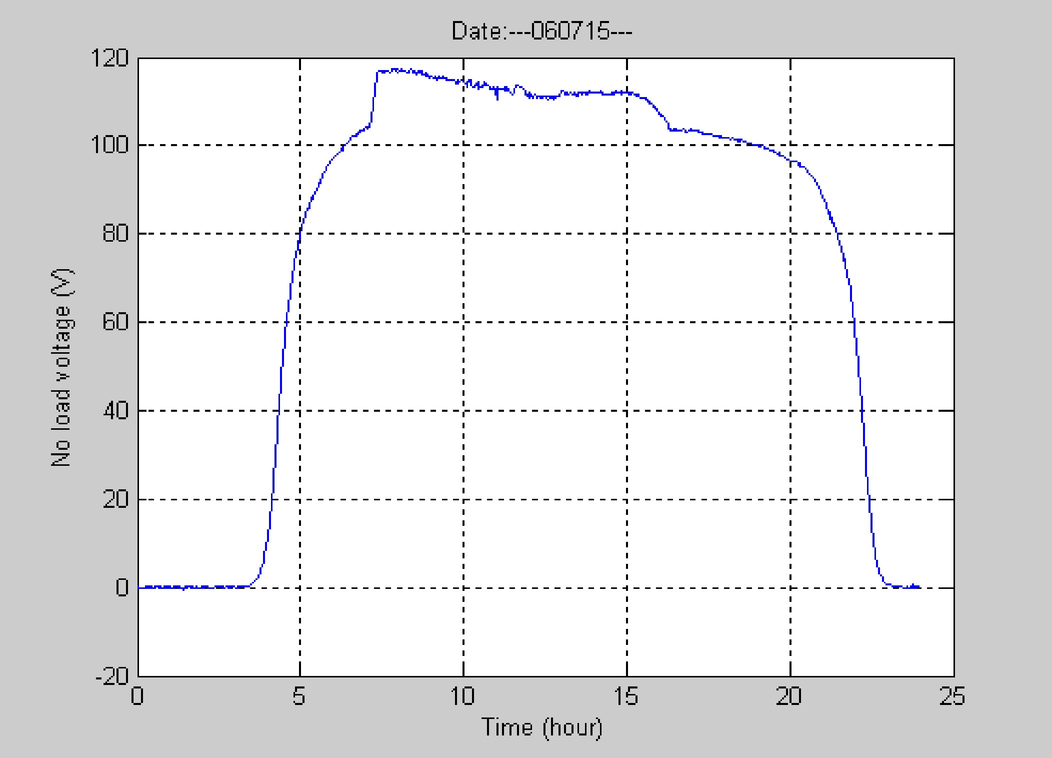

Figure 78 No load voltage 060715

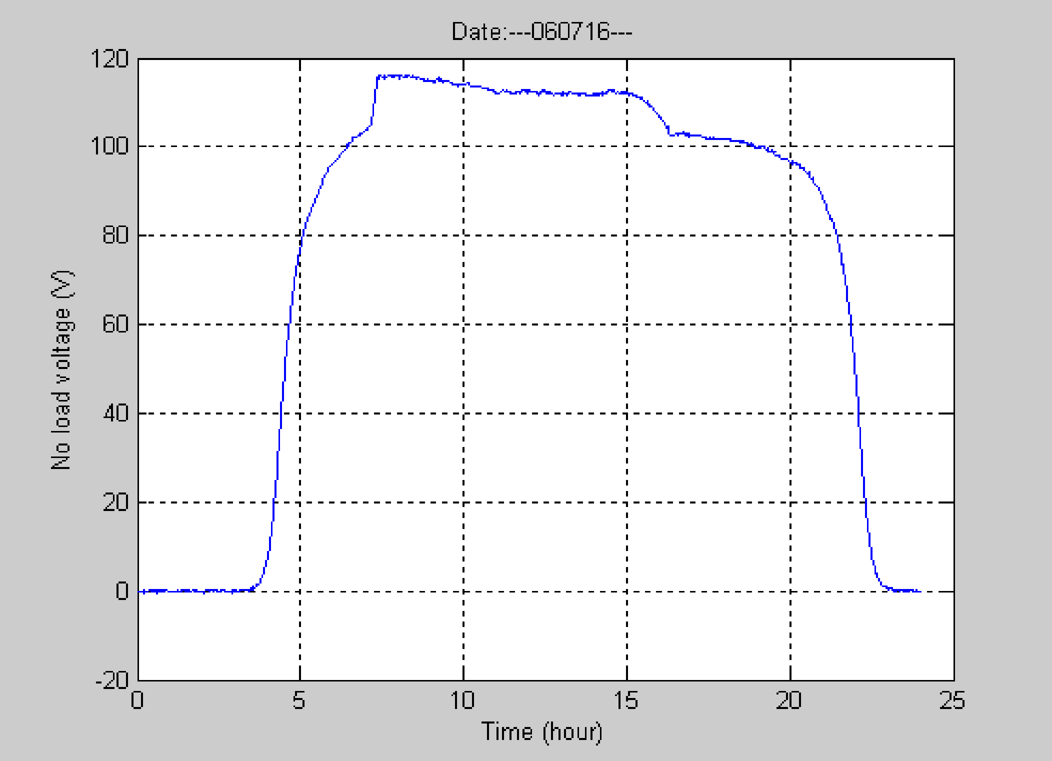

Figure 79 No load voltage 060716

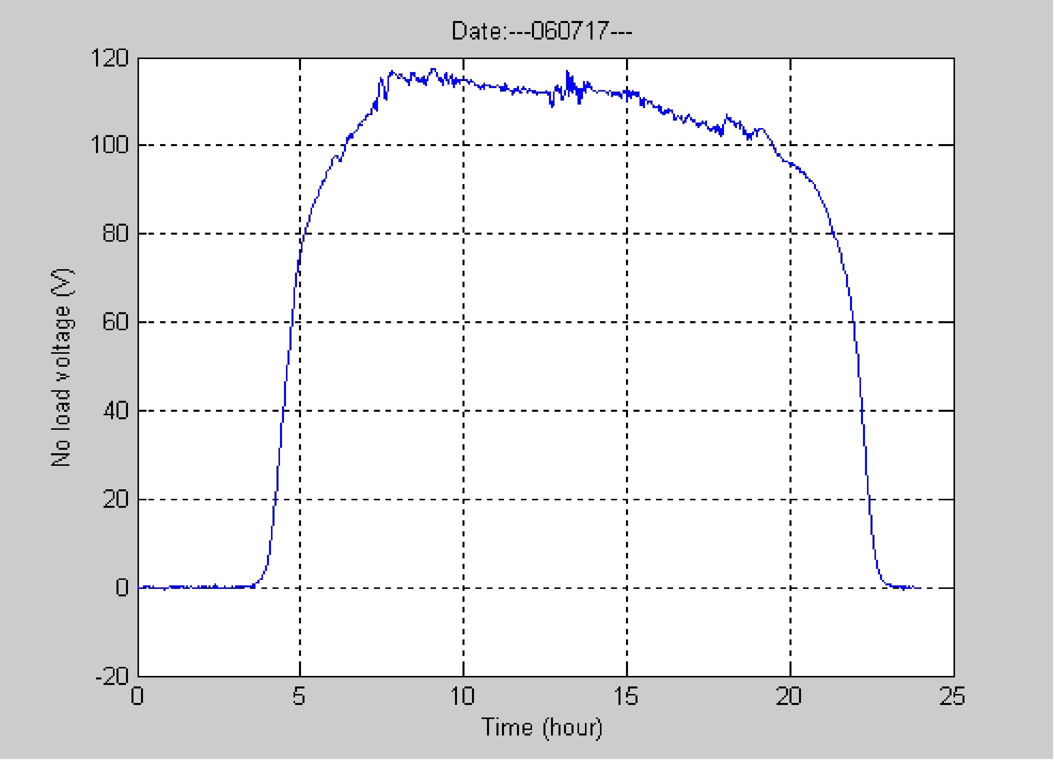

Figure 80 No load voltage 060717

5 ANALYSIS OF THE MEASUREMENTS

5.1 Common

The measurements consists of collected data in intervals of 1 minute during the period from 21/6 to 7/9 2006.

Figure 81 illustrates the extinction coefficient profile, based on the mesurements, during the period in question. 71 days in this period were used for "short circuit" measurements, giving basic data for extinction calculations. 4 days were used for "no load" measurements giving information about the top voltage profile. 4 days were not used for regular measurements.

In Figure 81 it could be noted that the "envelope" (the top values) of the extinction coefficient describes a falling curve, i.e. the maximum extinction is reduced during the time period in question.

Note. Days with extinction coefficient = 0 in Figure 81 represent “no load” measurements alternatively “no regular measurements”.

Figure 83 to Figure 112 illustrate some calculated values together with corresponding measured results from the 30 first measurement days. The solid curves (__) correspond to measured short circuit currents. The dashed (--) and dached-dotted (_ .) curves are calculated values. These calculations are based on Equation 11, where the dashed (--) curves corresponds to an extinction coefficient of 0.3126 and the dached-dotted (_ .) curves corresponds to the mean values of extinction coefficients between 9 am to 15 pm for the day in question. The value 0.3126 for the extinction coefficient corresponds to a value that for M = 1 and S_0 = 1367 W/m^2 gives S = 1000 W/m^2. See Equation 1, Equation 2 and Equation 3.

5.2 The extinction coefficients

The extinction coefficient is defined according to Equation 1. This is more and less a good approximation of the so called *“Beers law”*, that gives the transmission for the electromagnetic radiation in a very narrow wave length region in combination with a homogeneous transmission media. This is not on hand in the present application. So it must be pointed out that there is an approximation to use one single extinction coefficient to describe the transmission circumstances in this case. However the estimation is that the present approximation will result in a tool with a precision good enougt for statistic prediction of the potential to get electric power when using solar cells at different geographic locations and times of the year. Future validations will give more answeres about these questions.

The calculations of the extinction coefficients are based on Equation 10. The following parameters have then been used:

Idiff: The parameter corresponds to the resulted current component from the diffuse irradiance. The measured short circuited current at time point 16.30 has been used as value. The chosed time point will ensure that there is only diffuse radiation that hit the solar cells

I_M: The parameter corresponds to the short circuit current for a Sun irradiance of S_0 (Sun irradiance outside the atmosphere) and $\cos\beta$ = 1. This value has been predicted to 5 A

$\cos\beta$: The parameter corresponds to cosinus of the angle between the surface normal of the measuring surface (solar panel) and the direction to Sun. It is calculated as a result of the Sun altitude and azimuth and on the normal angle of the measuring surface relative to zenith and south. The Sun altitude and azimuth are calculated according to the description in [1]. The normal angle of the measuring surface relative to zenith is 90˚. The normal angle of the measuring surface relative to south is - 30˚. The equation to calculate $\cos\beta$ follows by Equation 5

M: The parameter corresponds to the relative atmospheric depth. It is calculated according to Equation 6 and the description in [1]

The mean values and standard deviations of the extinction coefficients between 9 am to 15 pm (Swedish summer time) for the first 30 short circuit measurement days are given in Table 1.

Measuring day (number)	Date	Cloudiness (mean value)	Mean value of extinction coefficient 9 am to 15 pm	Standard deviation of extinction coefficient 9 am to 15 pm
1	2006-06-21	5 – 6/8	1.82	1.12
2	-22	5 – 6/8	1.58	1.18
3	-23	5 – 6/8	2.03	1.09
4	-24	3 – 4/8	0.94	0.70
5	-25	5 – 6/8	1.98	0.62
6	-26	5 – 6/8	2.08	0.50
7	-27	7 – 8/8	2.42	0.49
8	-28	5 – 6/8	1.96	1.56
9	-29	1 – 2/8	0.65	0.56
10	-30	3 – 4/8	0.99	0.79
11	-07-01	0 – 1/8	0.44	0.04
12	-07-02	0 – 1/8	0.46	0.17
13	-07-03	0 – 1/8	0.46	0.05
14	-07-04	0 – 1/8	0.48	0.05
15	-07-05	0 – 1/8	0.50	0.04
16	-07-06	0 – 1/8	0.48	0.02

17	-07-07	7 – 8/8	2.27	1.54
18	-07-08	5 – 6/8	1.96	1.15
19	-07-09	5 – 6/8	1.72	1.00
20	-07-10	5 – 6/8	1.84	1.06
28	-07-18	0 – 1/8	0.42	0.01
29	-07-19	0 – 1/8	0.42	0.01
30	-07-20	0 – 1/8	0.41	0.12
31	-07-21	7 – 8/8	3.85	0.75
32	-07-22	7 – 8/8	2.42	1.42
33	-07-23	5 – 6/8	1.60	1.10
34	-07-24	5 – 6/8	1.99	1.22
35	-07-25	5 – 6/8	1.49	0.96
36	-07-26	0 – 1/8	0.44	0.02
37	-07-27	0 – 1/8	0.53	0.08
		Mean value of mean values:	1.35	0.65

Table 1 Mean values and standard deviations of extinction koefficients during 9 am to 15 pm for the first 30 measuring days of short circuited current

5.2.1 Probability function of the extinction coefficients

The following analysis presumes that the extinction coefficients are distributed according a so called *lognormal* distribution function. That implies that the natural logarithm of the extinction coefficients are normal distributed.

The mean values (μ) and standard deviations (σ) of the natural logarithm of the extinction coefficients between 9 am to 15 pm (Swedish summer time) for the first 30 short circuit measurement days are given in Table 2.

The total mean value of the statistical parameters (μ and σ) in Table 2 is

μtot = -0.8863 and σtot = 0.9652

Presuming these values gives the lognormal density function according to Figure 82.

Measuring day (number)	Date	Cloudiness (mean value)	Mean value of nat log for extinction coefficient 9 am to 15 pm **μ**	Standard deviation of nat log for extinction coefficient 9 am to 15 pm **σ**
1	2006-06-21	5 – 6/8	0.0602	1.0661
2	-22	5 – 6/8	-0.5869	1.6811
3	-23	5 – 6/8	-0.1390	2.2193
4	-24	3 – 4/8	-1.0329	1.4186
5	-25	5 – 6/8	0.4247	0.4587
6	-26	5 – 6/8	0.5209	0.3211
7	-27	7 – 8/8	0.7211	0.2264
8	-28	5 – 6/8	-0.8158	2.5341
9	-29	1 – 2/8	-2.0478	1.4178
10	-30	3 – 4/8	-1.8986	2.8009
11	-07-01	0 – 1/8	-2.0920	0.2162
12	-07-02	0 – 1/8	-2.0284	0.3451
13	-07-03	0 – 1/8	-1.9814	0.2419
14	-07-04	0 – 1/8	-1.8449	0.2668
15	-07-05	0 – 1/8	-1.6916	0.2056
16	-07-06	0 – 1/8	-1.7998	0.1313

17	-07-07	7 – 8/8	0.1217	1.2163
18	-07-08	5 – 6/8	0.0141	1.3137
19	-07-09	5 – 6/8	0.0854	0.7732
20	-07-10	5 – 6/8	-0.1951	1.7054
28	-07-18	0 – 1/8	-2.2442	0.0905
29	-07-19	0 – 1/8	-2.2309	0.0792
30	-07-20	0 – 1/8	-2.5169	0.8713
31	-07-21	7 – 8/8	1.2291	0.2951
32	-07-22	7 – 8/8	0.2460	1.3772
33	-07-23	5 – 6/8	-0.3687	1.5807
34	-07-24	5 – 6/8	-0.1083	1.5530
35	-07-25	5 – 6/8	-0.7527	2.1293
36	-07-26	0 – 1/8	-2.0537	0.1183
37	-07-27	0 – 1/8	-1.5838	0.3016
		Mean value of mean values:	-0.8863	0.9652

Table 2 Mean values and standard deviations of the natural logarithm for extinction coefficients during 9 am to 15 pm for the first 30 measuring days of short circuited current

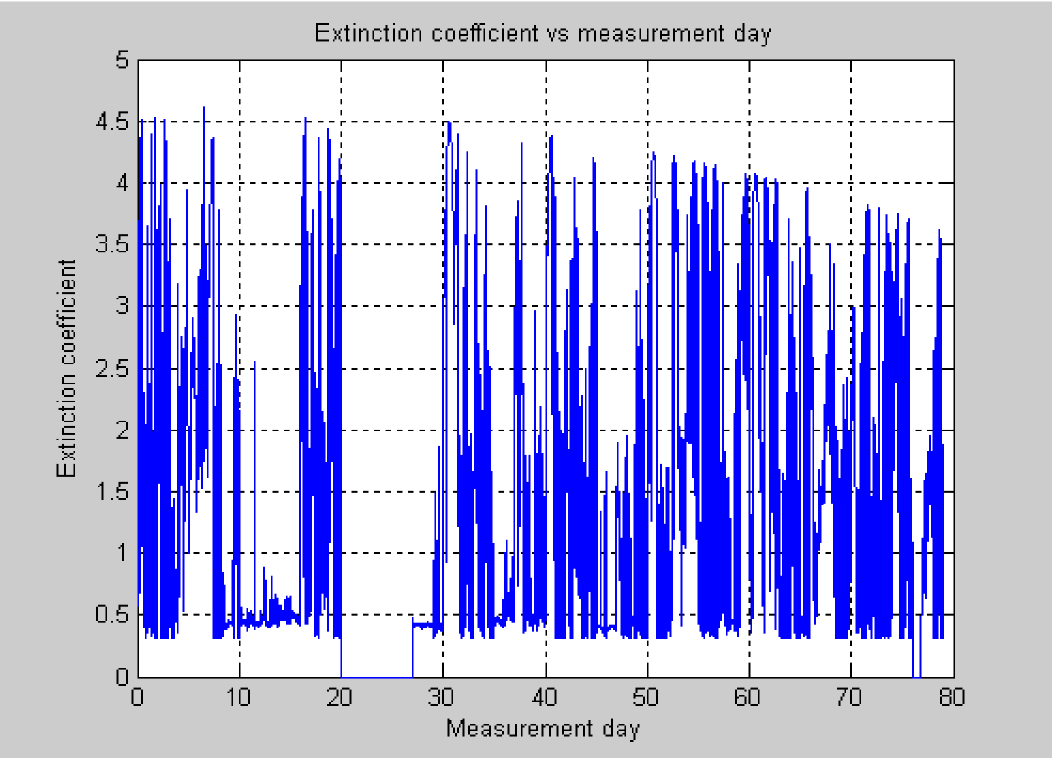

Figure 81 Profile of the extinction coefficient, based on the mesurements, during the period from 21/6 to 7/9 2006.

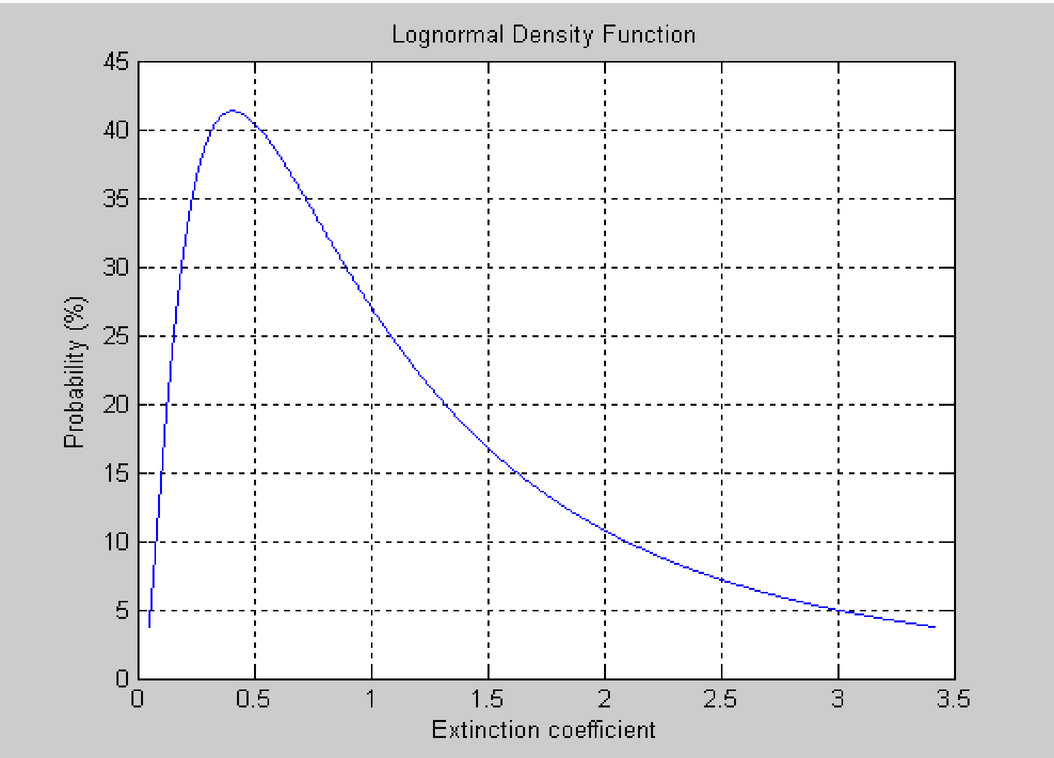

Figure 82 The extinction coefficient density function with μtot = -0.8863 and σtot = 0.9652

Figure 83 to Figure 112 illustrate some calculated short circuit currents together with corresponding measured results from the 30 first measurement days. The solid curves (__) correspond to measured values. The dashed (--) and dached-dotted (_ .) curves are calculated values with different extinction koefficients. The calculations are based on Equation 11.

The dashed (--) curves are based on the extinction coefficient = 0.3126. This value has been assumed as the under limit of a realistic extinction coefficient. The value is based on calculations according to Equation 1, Equation 2 and Equation 3, with the following parameters:

M: 1 (corresponding to the Sun in zenith)

S: 1000 W/m^2

S_0: 1367 W/m^2

The dached-dotted (_ .) curves are based on the extinction coefficients according to the mean value of extinction coefficient between 9 am to 15 pm for the measurement day in question. See Table 1.

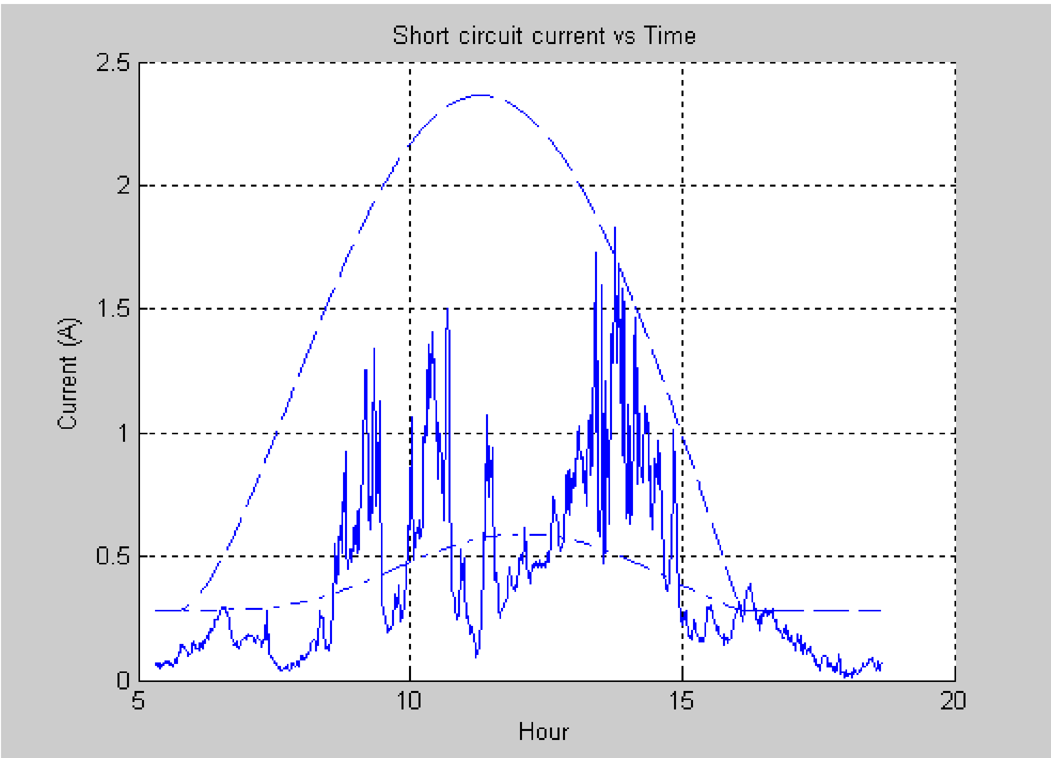

Figure 83 Measured and calculated short circuit current 060621
____: measured, _ _: 0.3126, _ . _: 1.8242

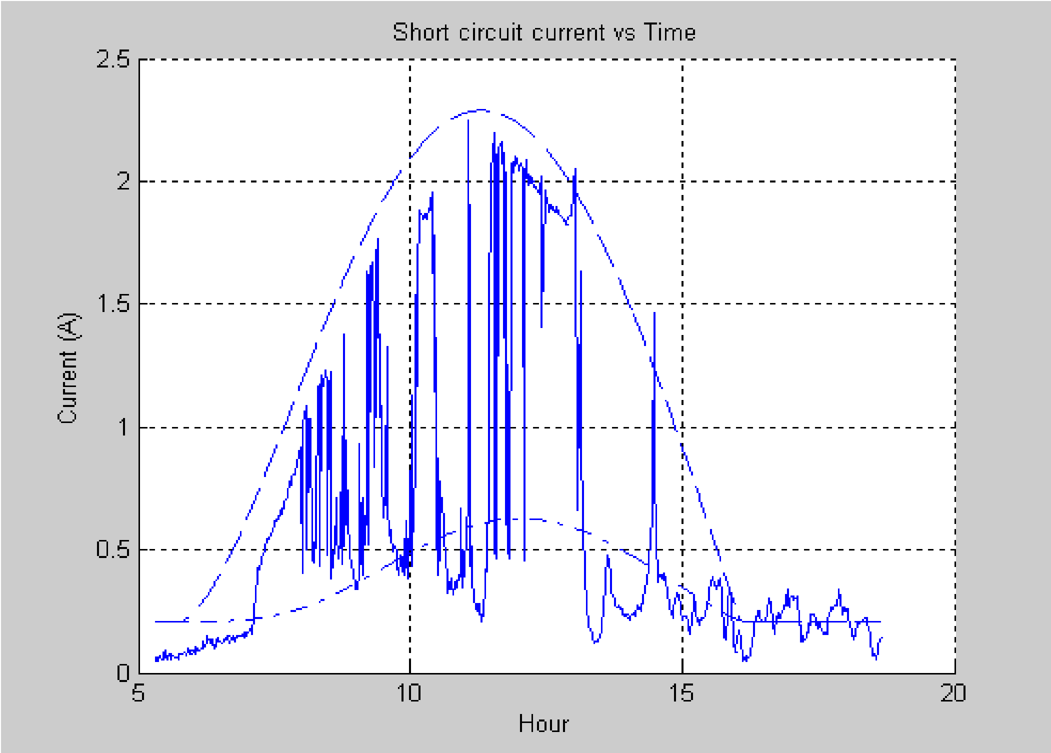

Figure 84 Measured and calculated short circuit current 060622
_____: measured, _ _: 0.3126, _ . _: 1.5760

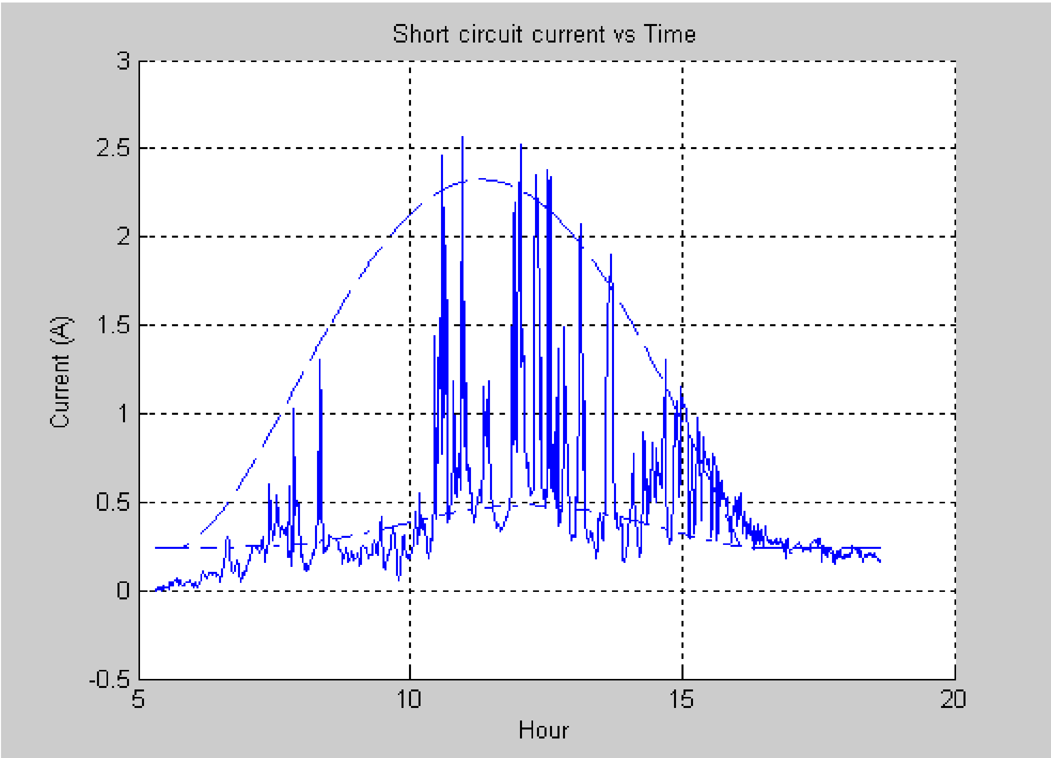

Figure 85 Measured and calculated short circuit current 060623
_____: measured, _ _: 0.3126, _ . _: 2.0317

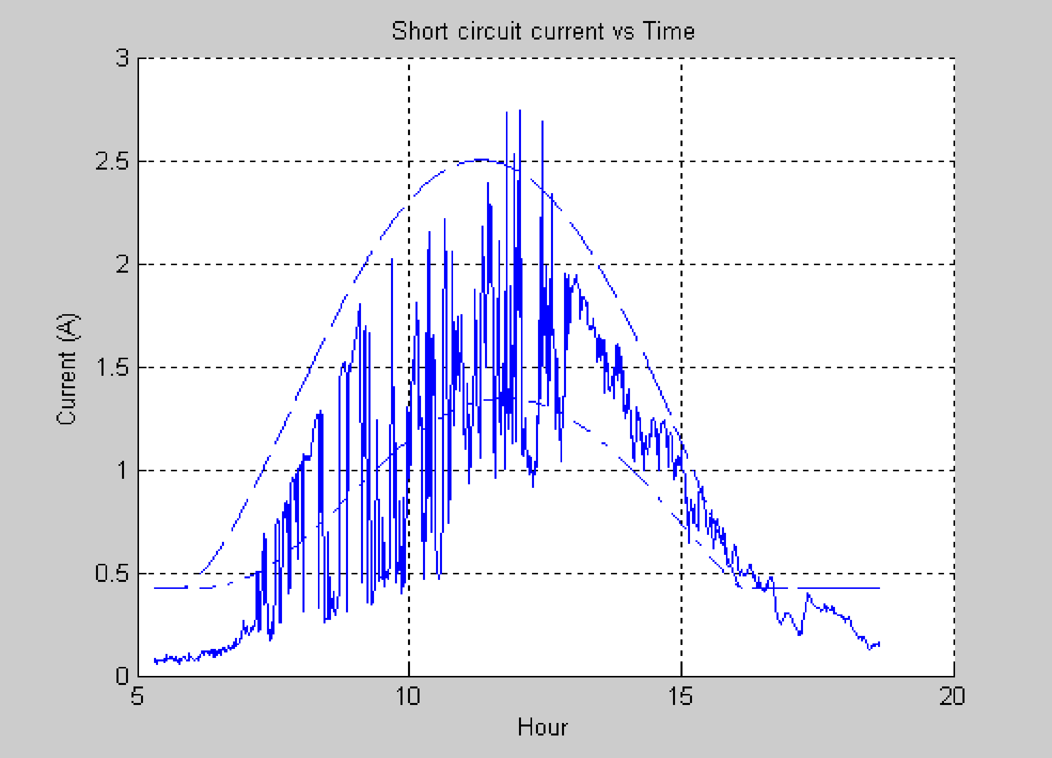

Figure 86 Measured and calculated short circuit current 060624
____: measured, _ _: 0.3126, _ . _: 0.9449

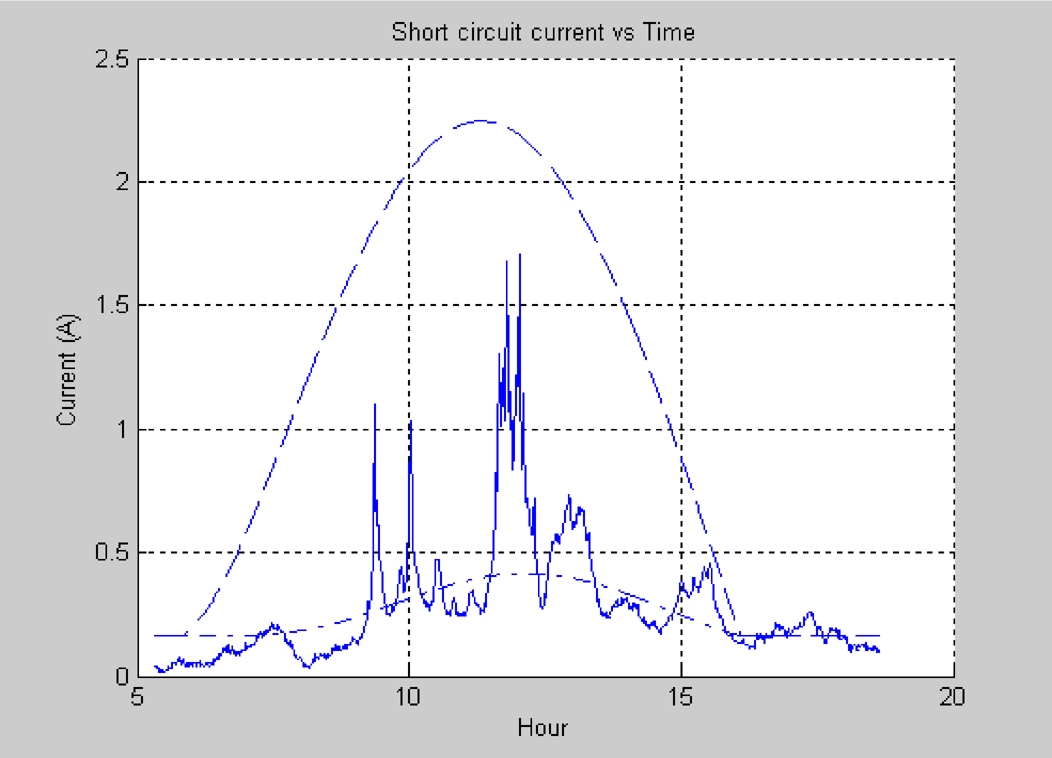

Figure 87 Measured and calculated short circuit current 060625
____: measured, _ _: 0.3126, _ . _: 1.9826

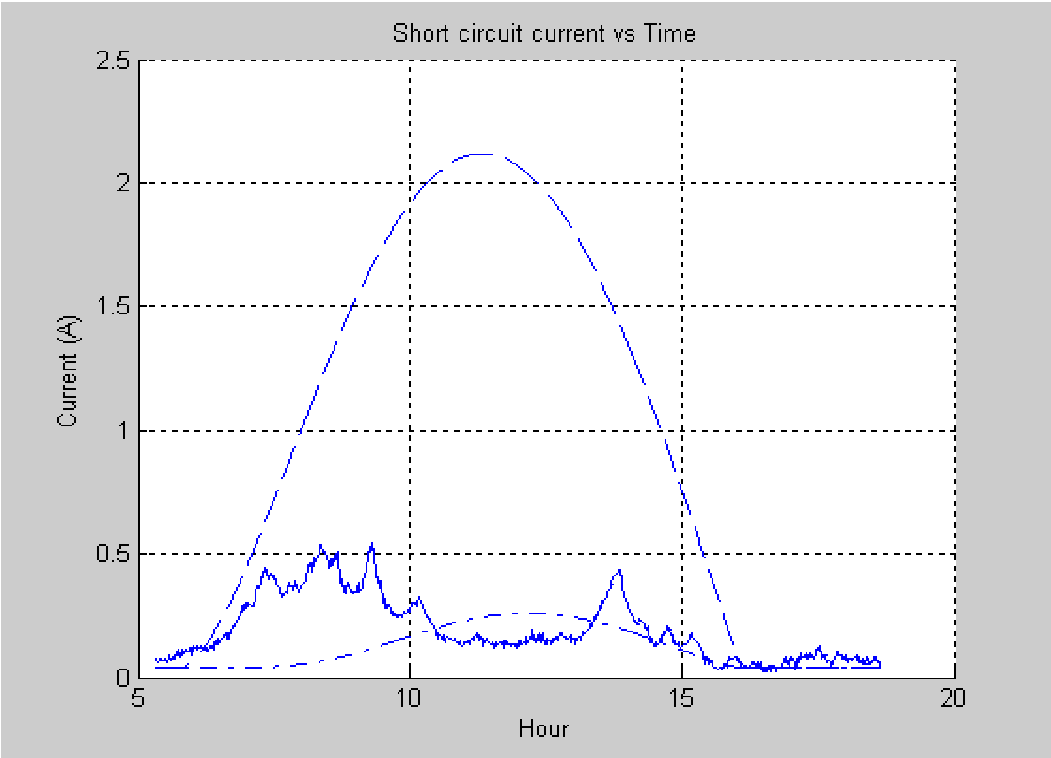

Figure 88 Measured and calculated short circuit current 060626
_____: measured, _ _: 0.3126, _ . _: 2.0771

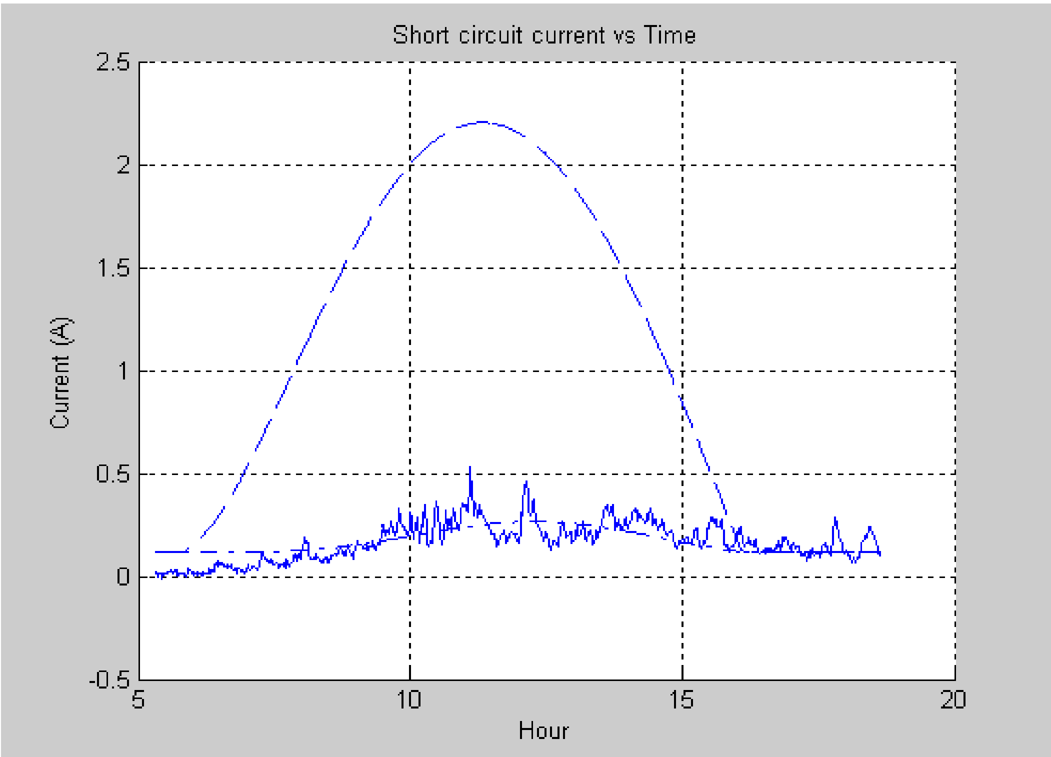

Figure 89 Measured and calculated short circuit current 060627
_____: measured, _ _: 0.3126, _ . _: 2.4233

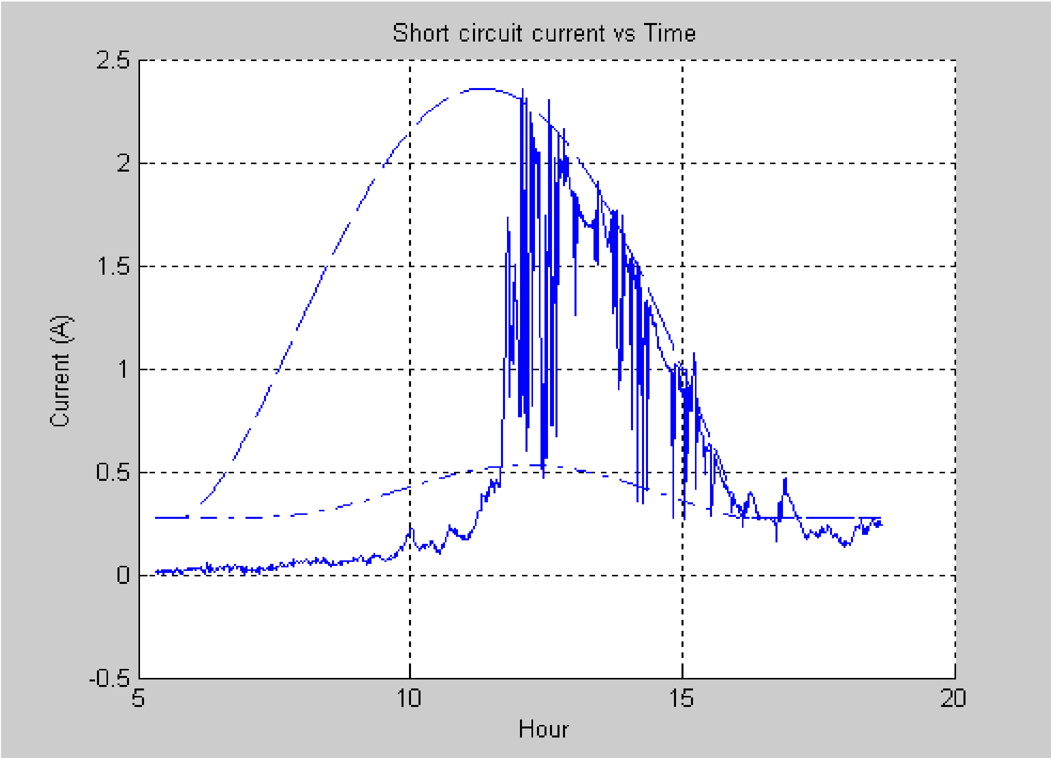

Figure 90	Measured and calculated short circuit current 060628
____: measured, _ _: 0.3126, _ . _: 1.9638

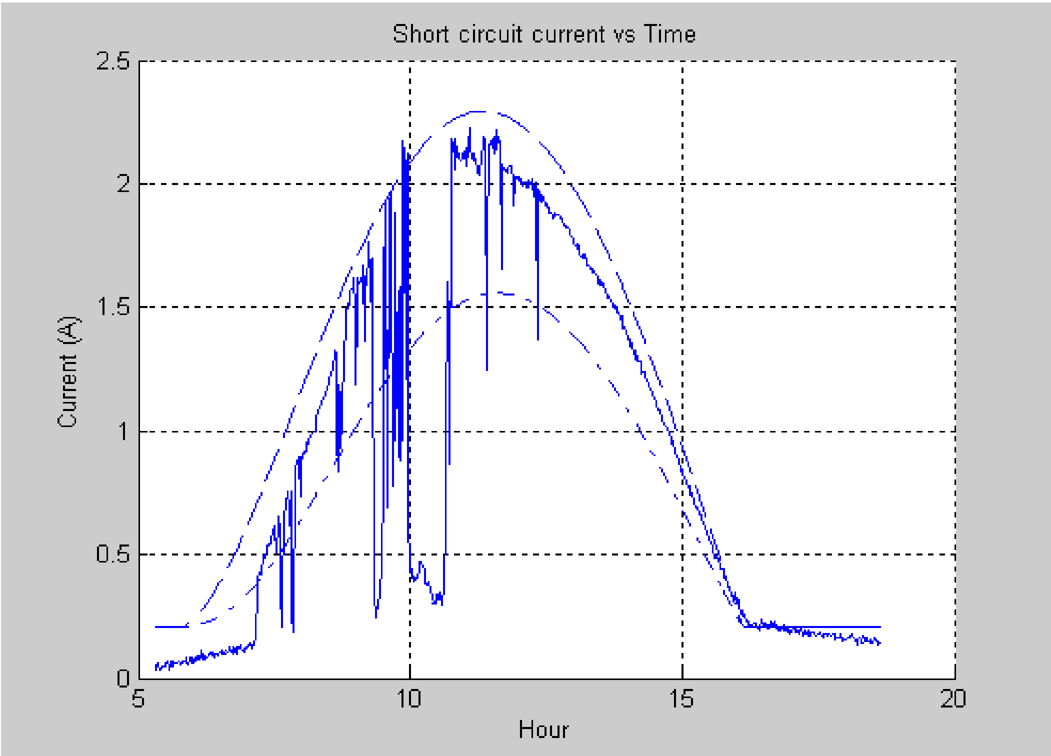

Figure 91	Measured and calculated short circuit current 060629
____: measured, _ _: 0.3126, _ . _: 0.6486

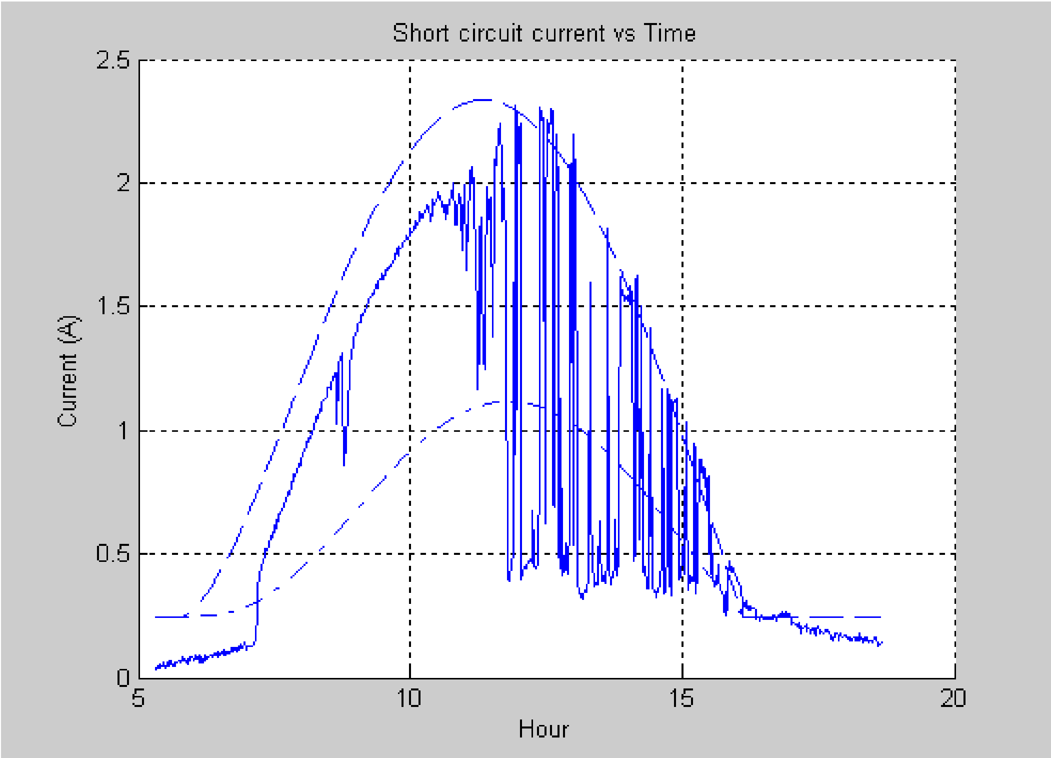

Figure 92 Measured and calculated short circuit current 060630
____: measured, _ _: 0.3126, _ . _: 0.9933

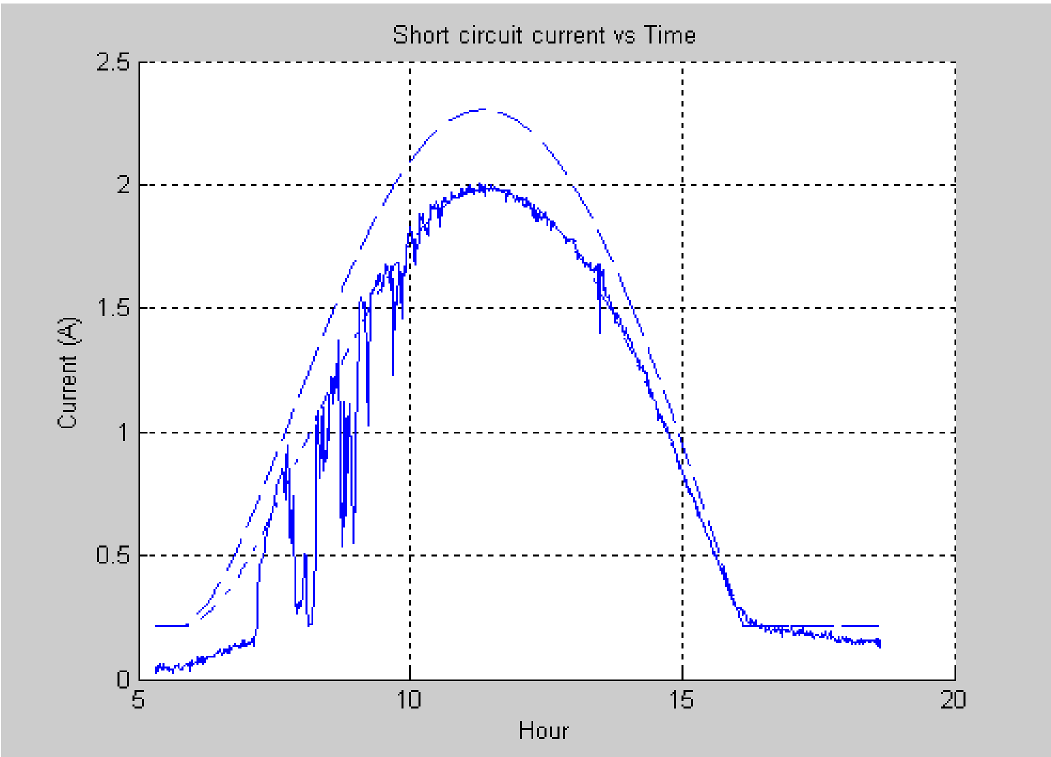

Figure 93 Measured and calculated short circuit current 060701
____: measured, _ _: 0.3126, _ . _: 0.4393

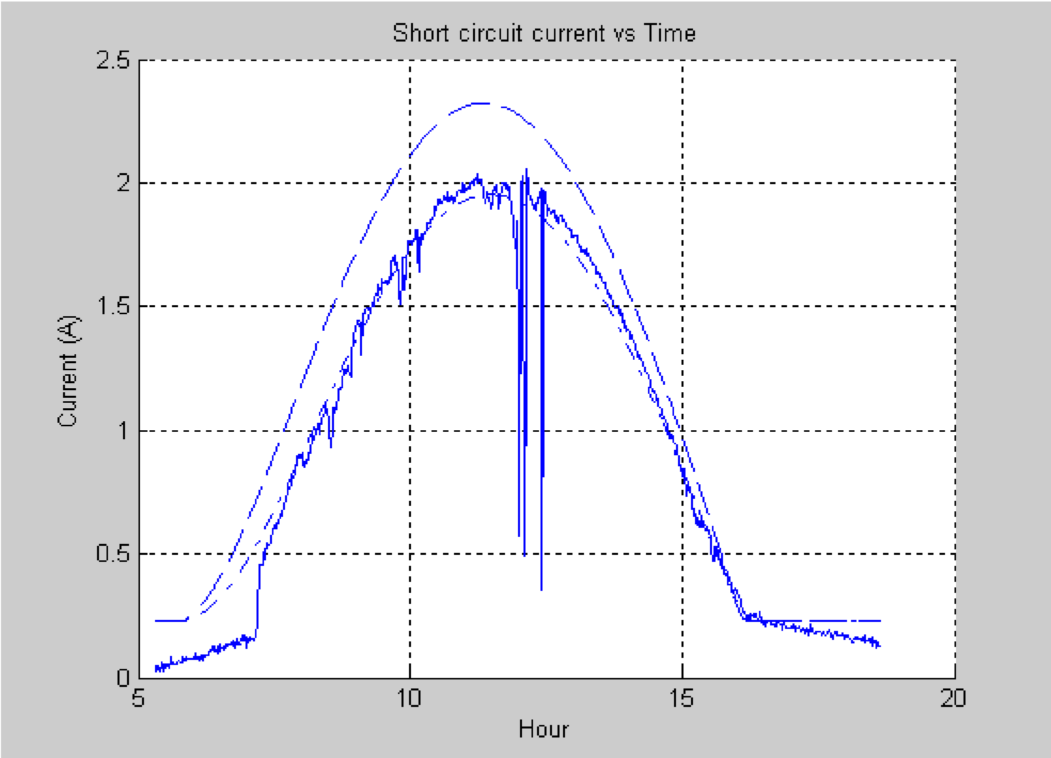

Figure 94 Measured and calculated short circuit current 060702
____: measured, _ _: 0.3126, _ . _: 0.4615

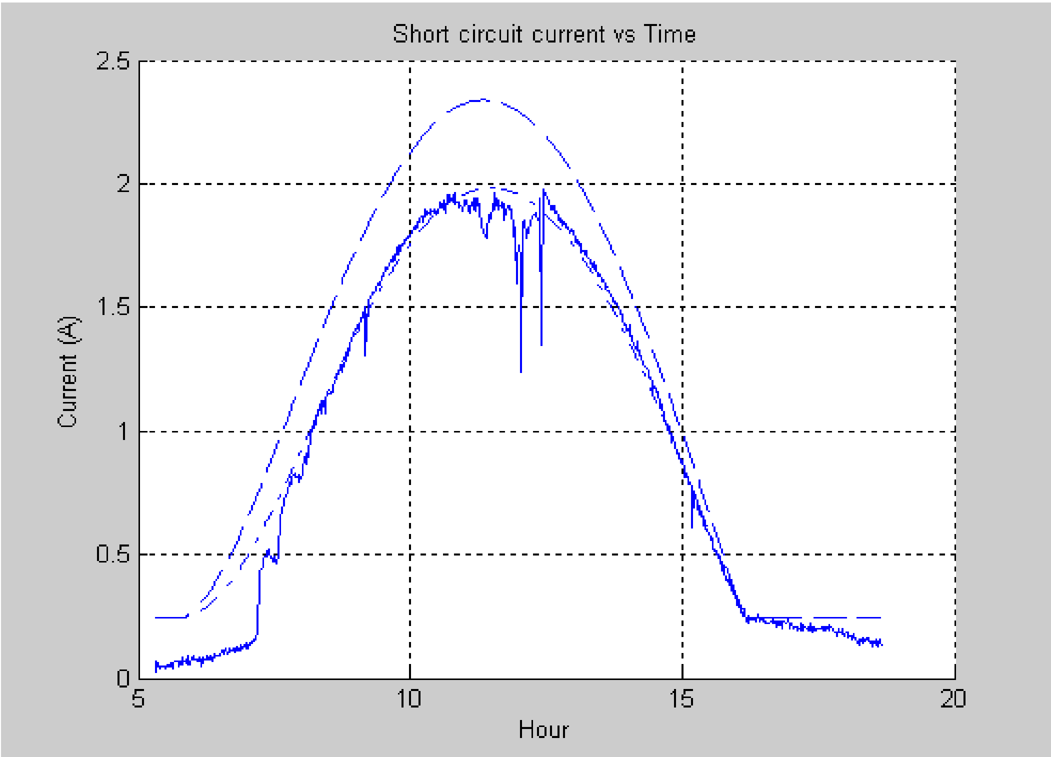

Figure 95 Measured and calculated short circuit current 060703
____: measured, _ _: 0.3126, _ . _: 0.4553

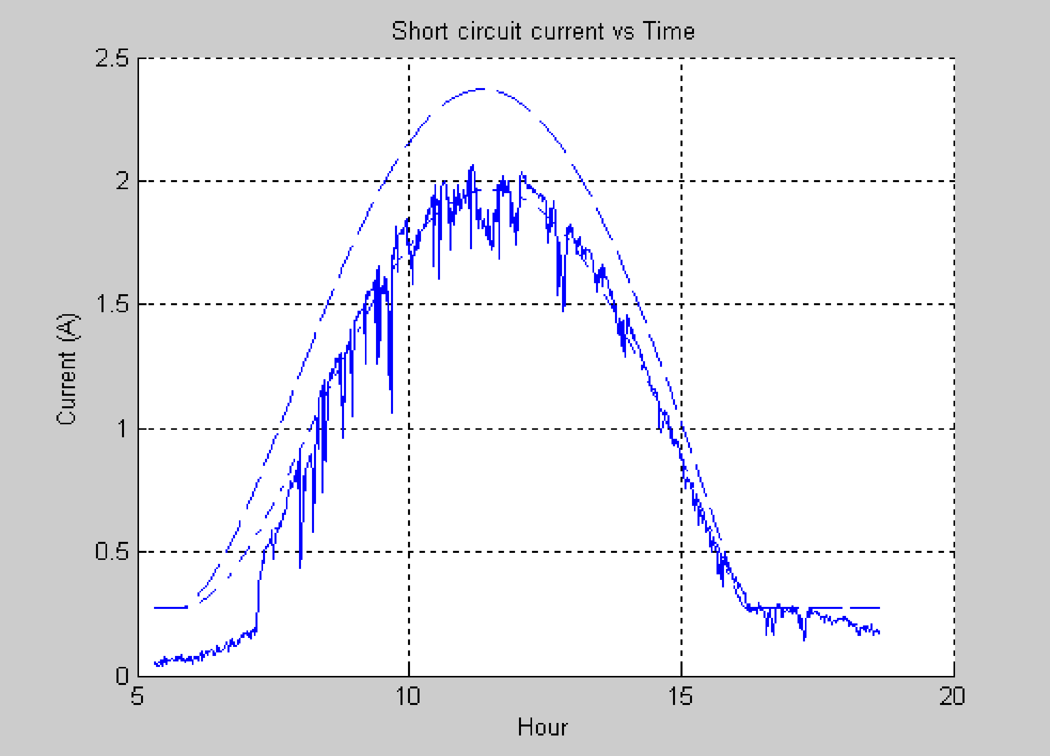

Figure 96 Measured and calculated short circuit current 060704
____: measured, _ _: 0.3126, _ . _: 0.4769

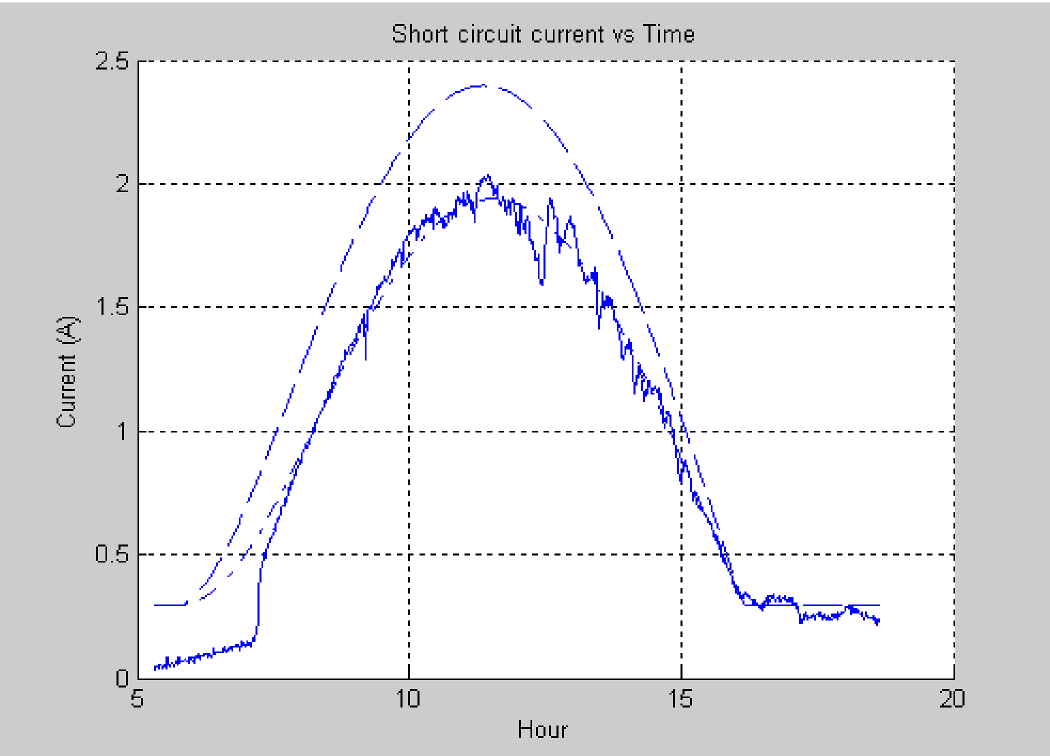

Figure 97 Measured and calculated short circuit current 060705
____: measured, _ _: 0.3126, _ . _: 0.5008

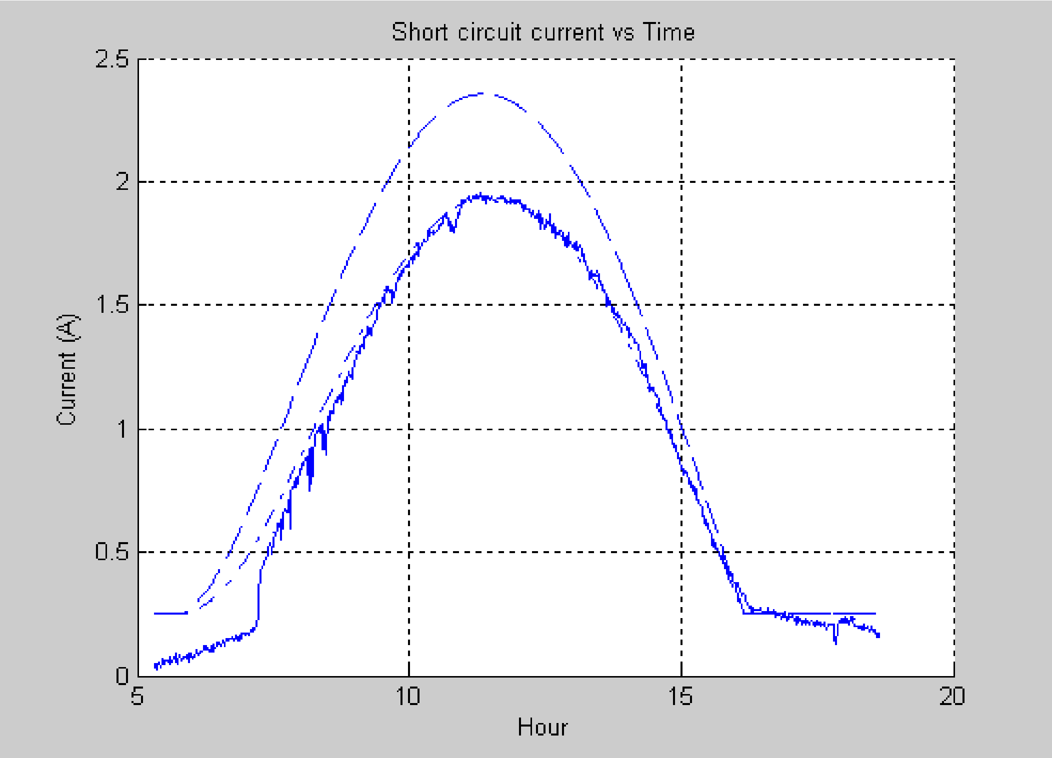

Figure 98 Measured and calculated short circuit current 060706
____: measured, _ _: 0.3126, _ . _: 0.4792

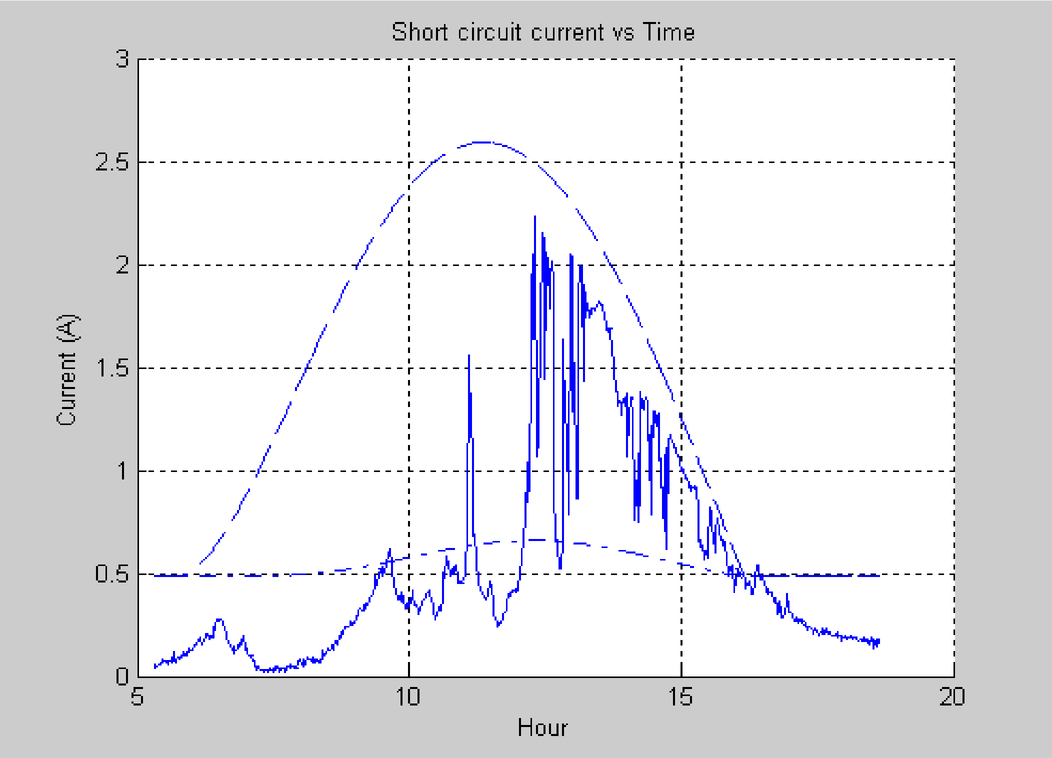

Figure 99 Measured and calculated short circuit current 060707
____: measured, _ _: 0.3126, _ . _: 2.2706

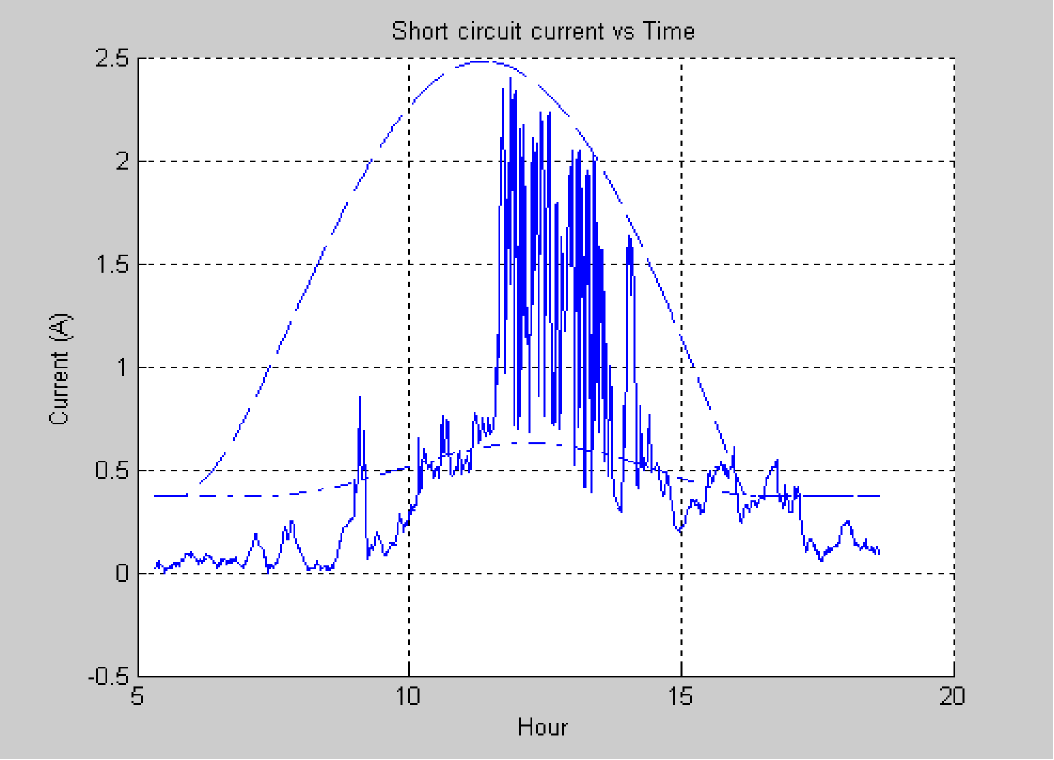

Figure 100 Measured and calculated short circuit current 060708
_____: measured, _ _: 0.3126, _ . _: 1.9575

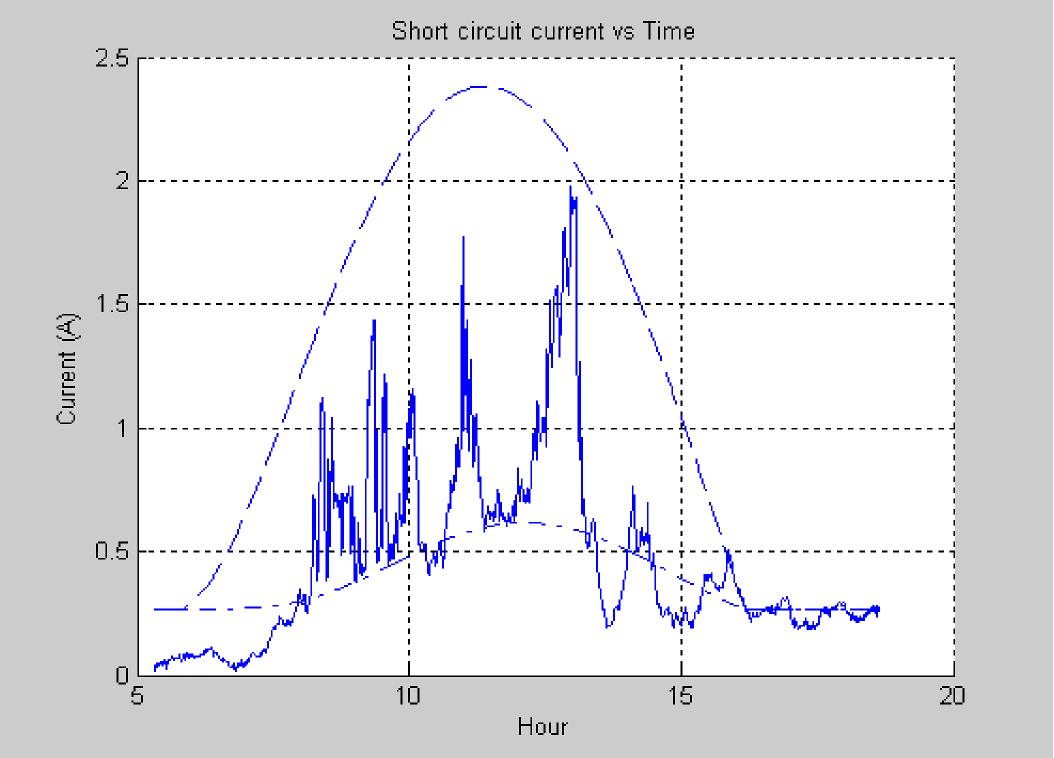

Figure 101 Measured and calculated short circuit current 060709
_____: measured, _ _: 0.3126, _ . _: 1.7162

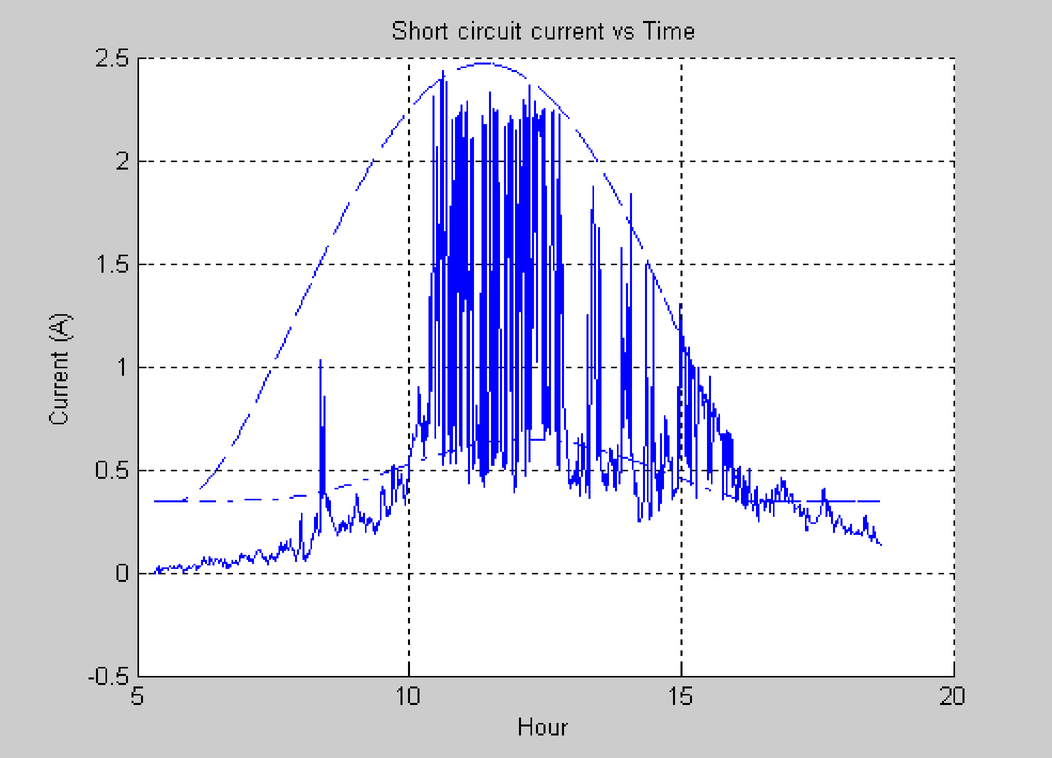

Figure 102 Measured and calculated short circuit current 060710
____: measured, _ _: 0.3126, _ . _: 1.8421

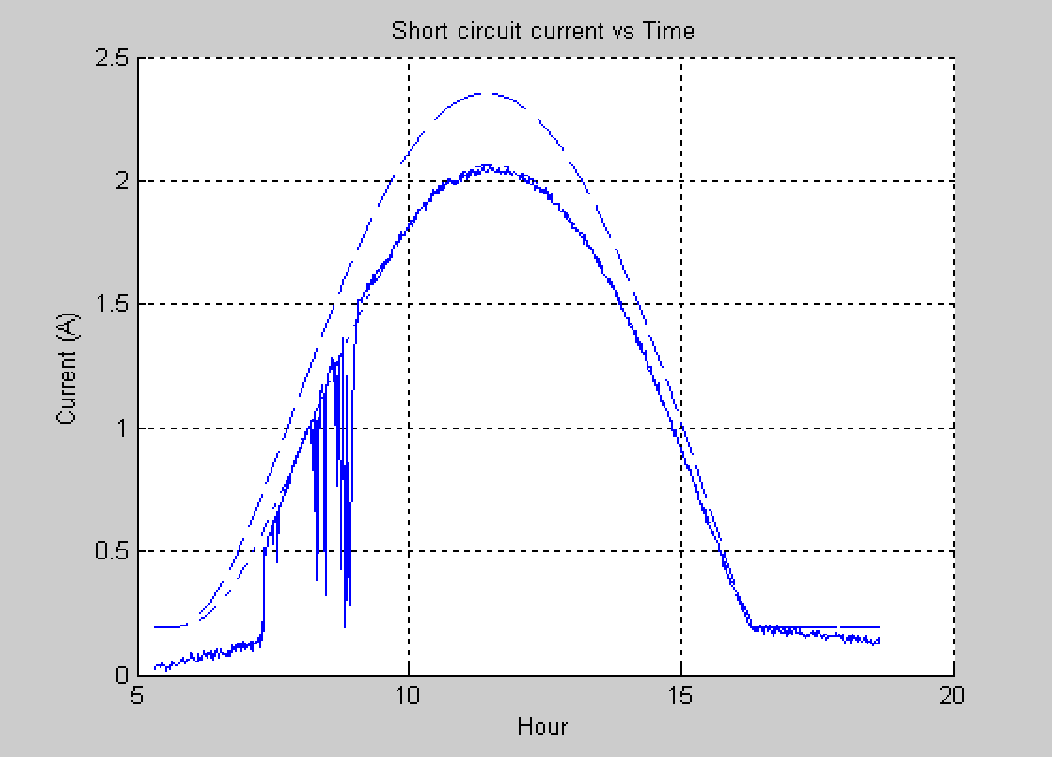

Figure 103 Measured and calculated short circuit current 060718
____: measured, _ _: 0.3126, _ . _: 0.4189

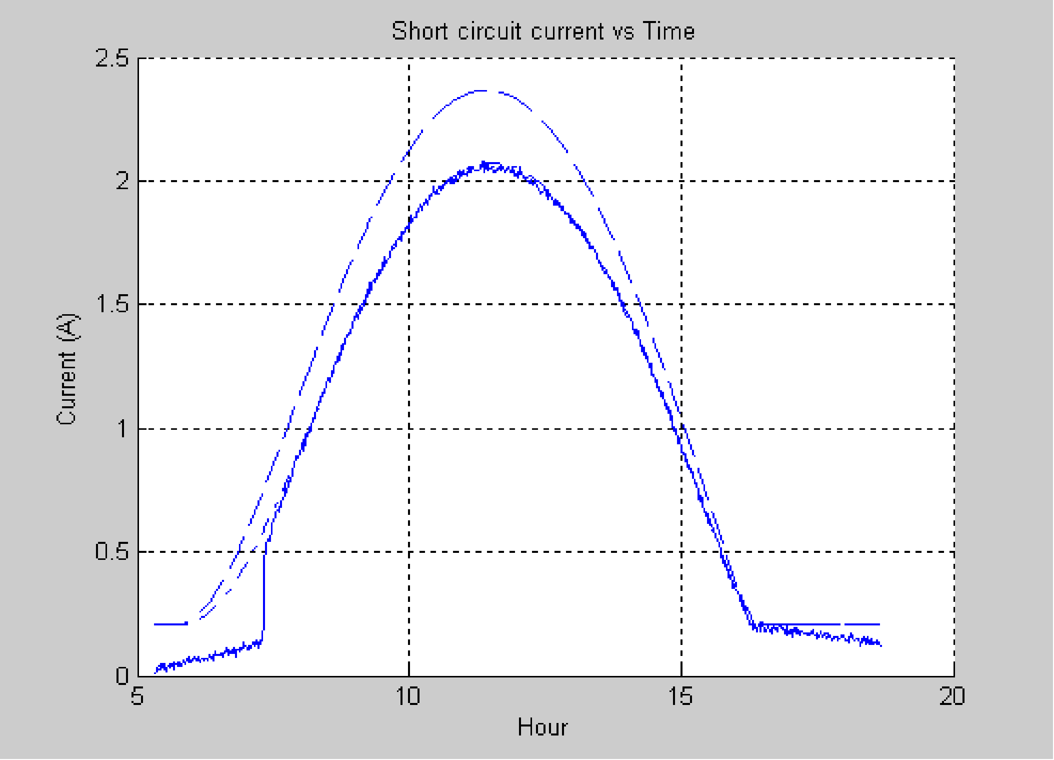

Figure 104 Measured and calculated short circuit current 060719
____: measured, _ _: 0.3126, _ . _: 0.4203

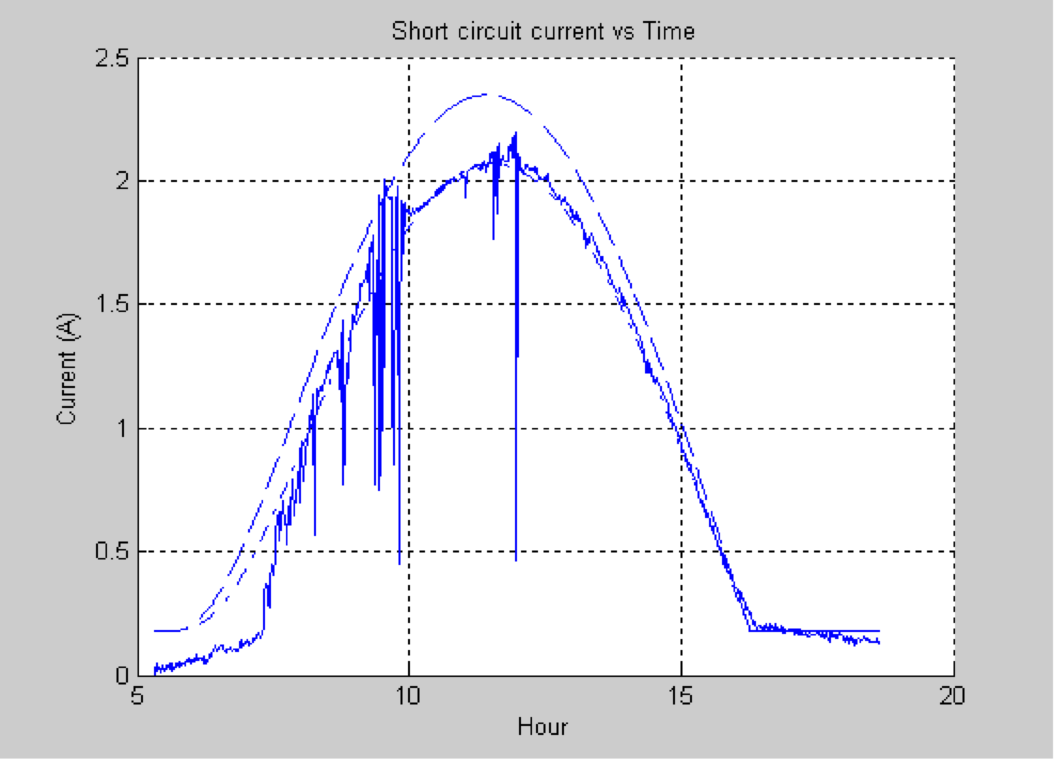

Figure 105 Measured and calculated short circuit current 060720
____: measured, _ _: 0.3126, _ . _: 0.4144

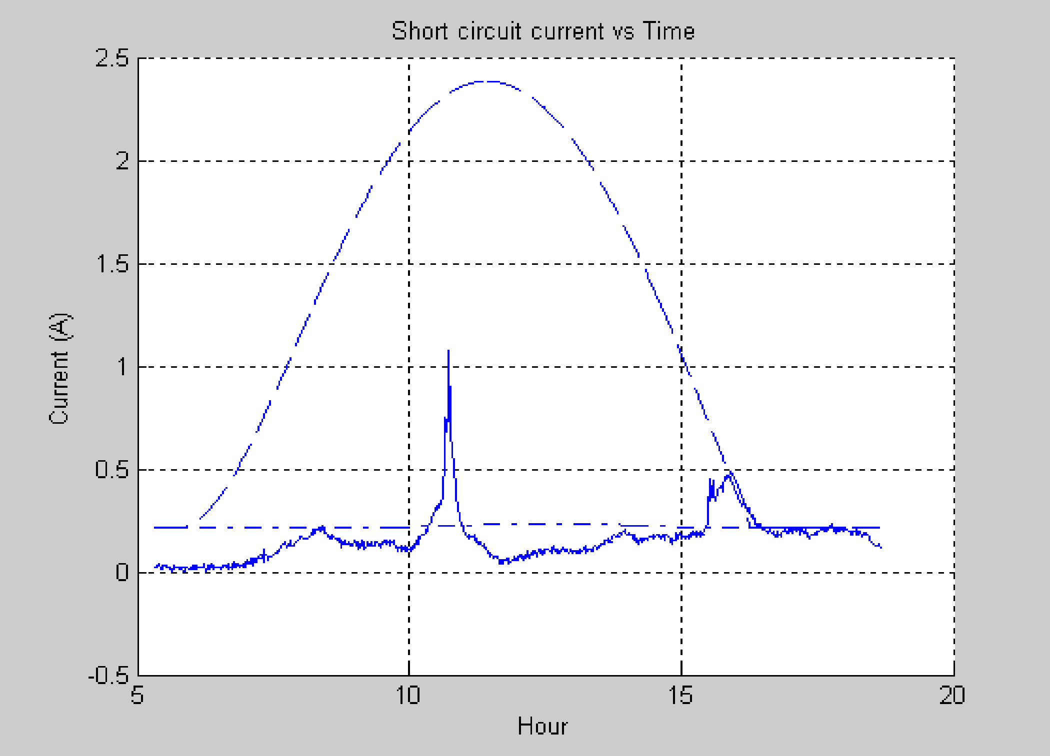

Figure 106 Measured and calculated short circuit current 060721
_____: measured, _ _: 0.3126, _ . _: 3.8493

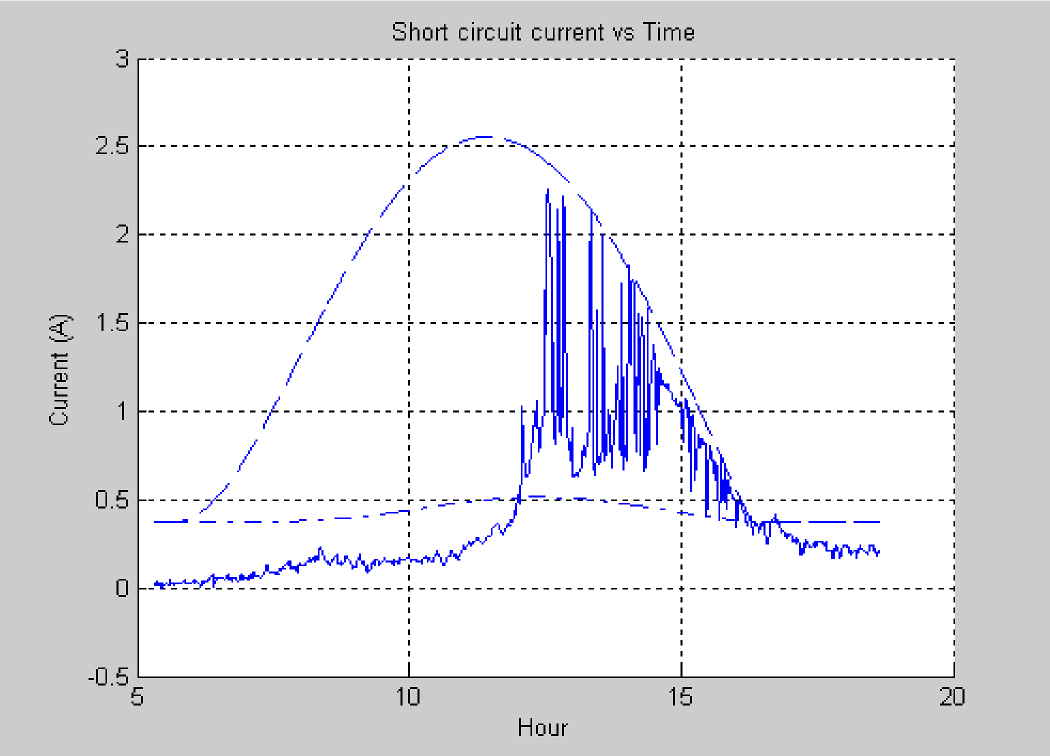

Figure 107 Measured and calculated short circuit current 060722
_____: measured, _ _: 0.3126, _ . _: 2.4203

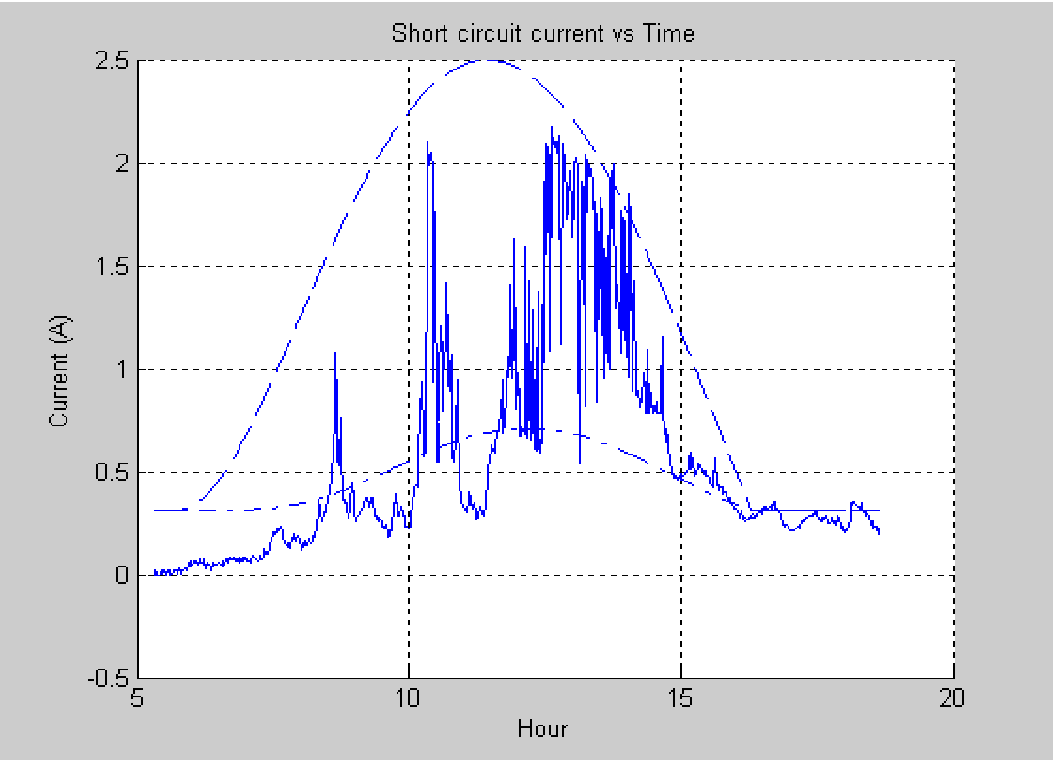

Figure 108 Measured and calculated short circuit current 060723
____: measured, _ _: 0.3126, _ . _: 1.6010

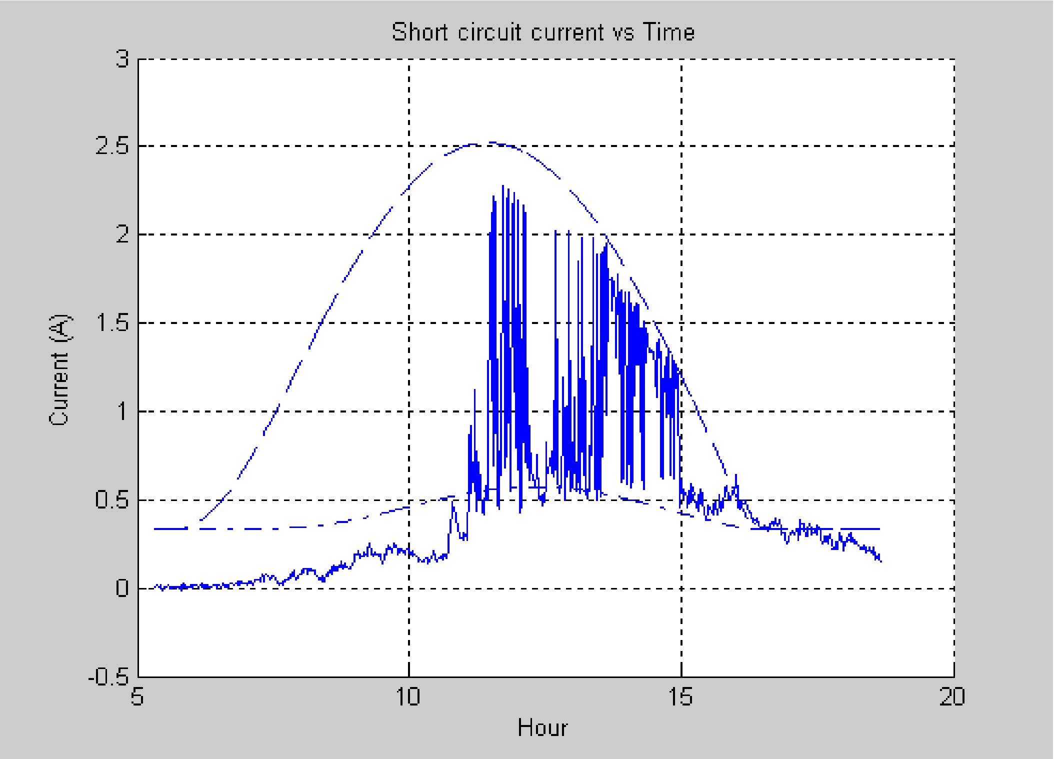

Figure 109 Measured and calculated short circuit current 060724
____: measured, _ _: 0.3126, _ . _: 1.9856

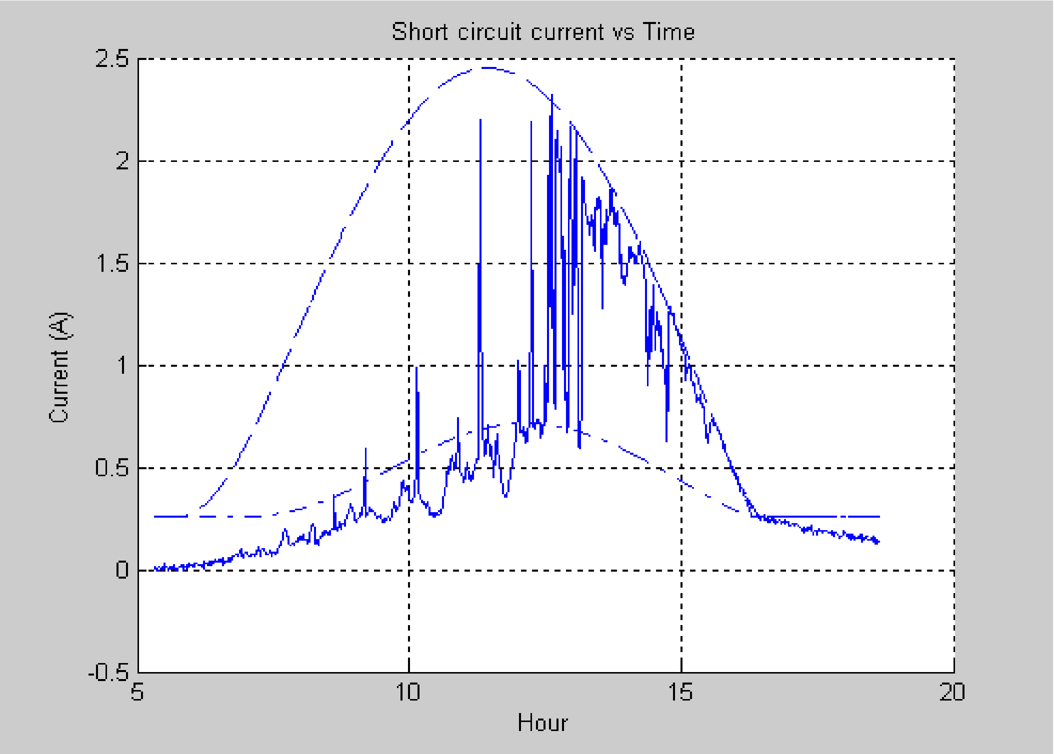

Figure 110 Measured and calculated short circuit current 060725
____: measured, _ _: 0.3126, _ . _: 1.4888

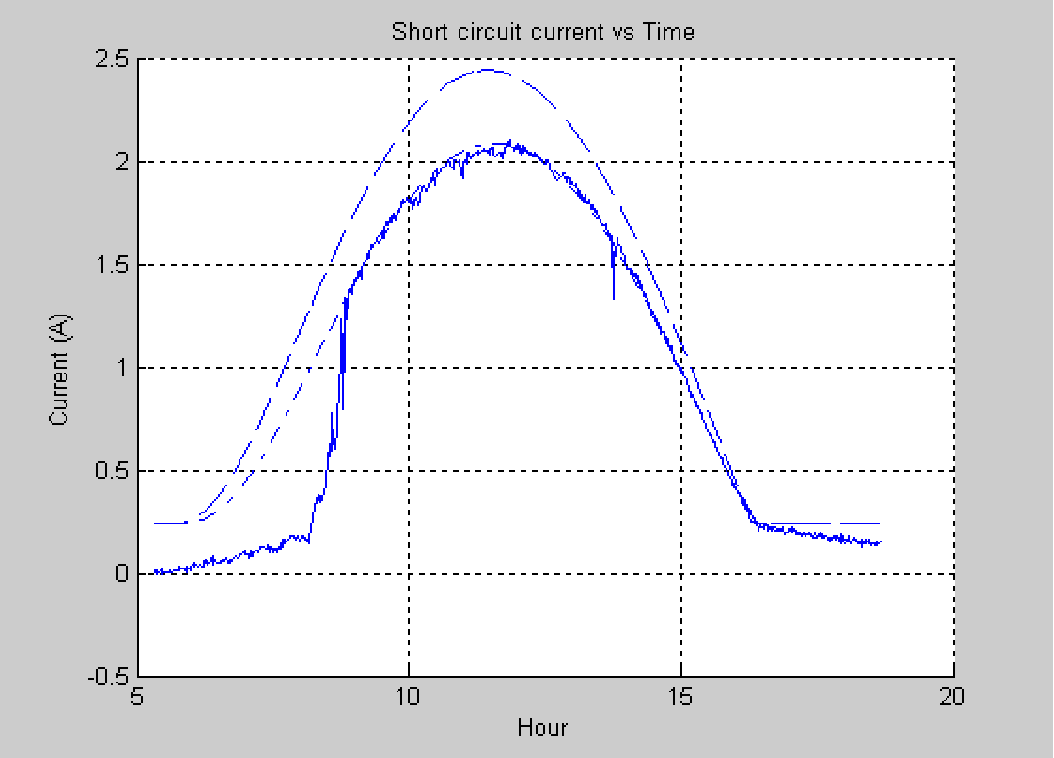

Figure 111 Measured and calculated short circuit current 060726
____: measured, _ _: 0.3126, _ . _: 0.4417

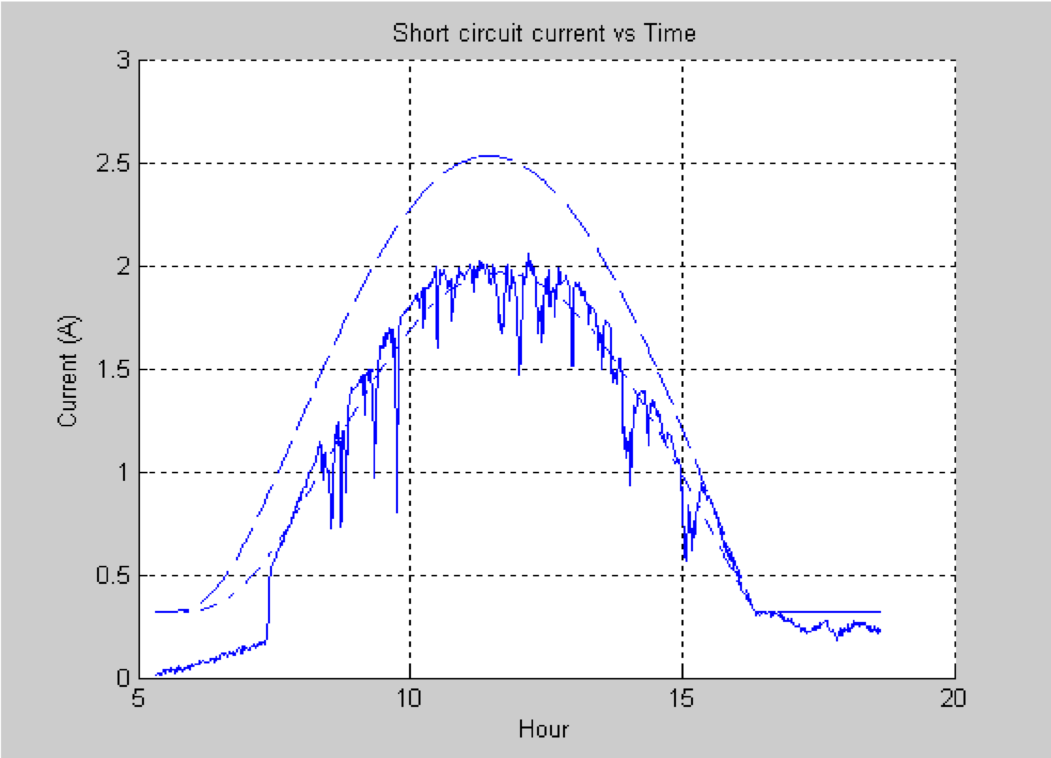

Figure 112 Measured and calculated short circuit current 060727
____: measured, _ _: 0.3126, _ . _: 0.5288

Figure 113 to Figure 121 give the relation between the extinction coefficient and the corresponding short circuit current for some measurement days. As can be observed is for example that a relative high extinction coeffient (low atmospheric transmission) results in a low short circuit current. A low extinction coefficient (high atmospheric transmission) results in a high short circuit current.

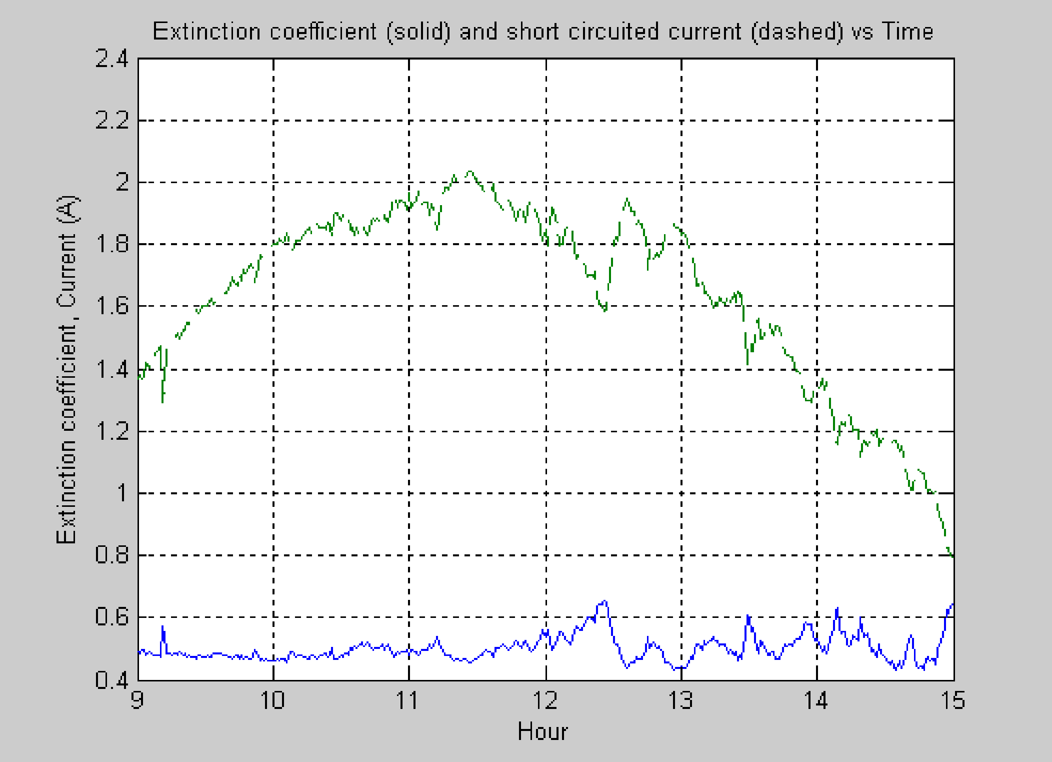

Figure 113 Extinction coefficient and short circuited current vs Time. 060705

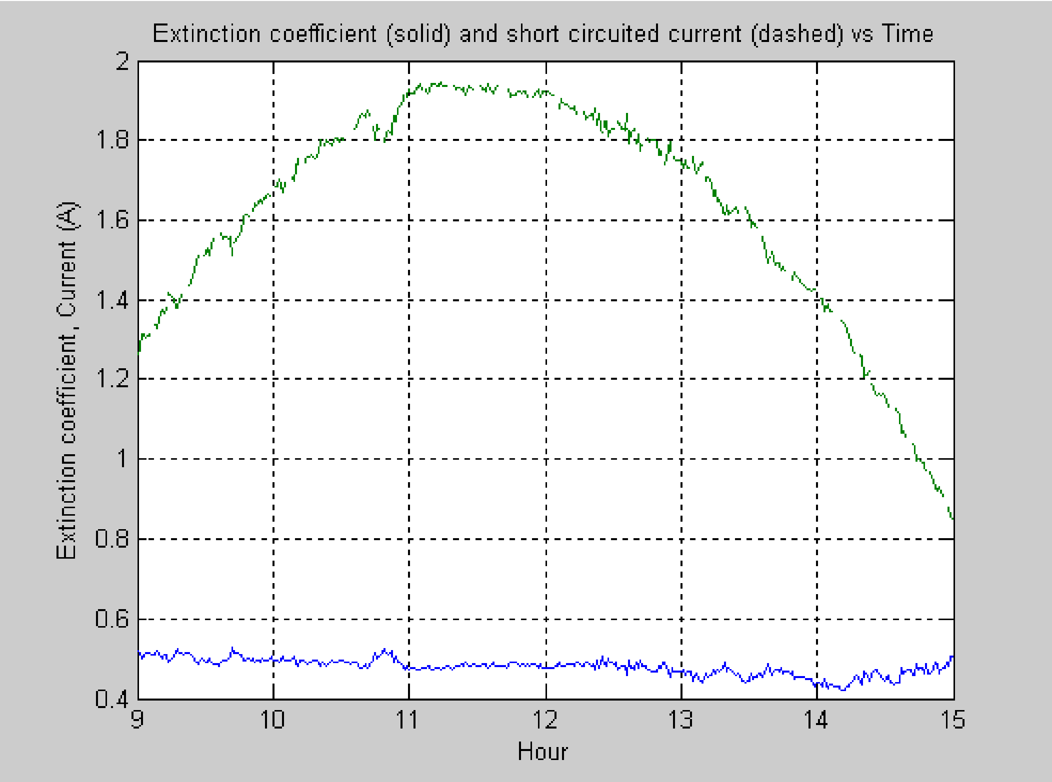

Figure 114 Extinction coefficient and short circuited current vs Time. 060706

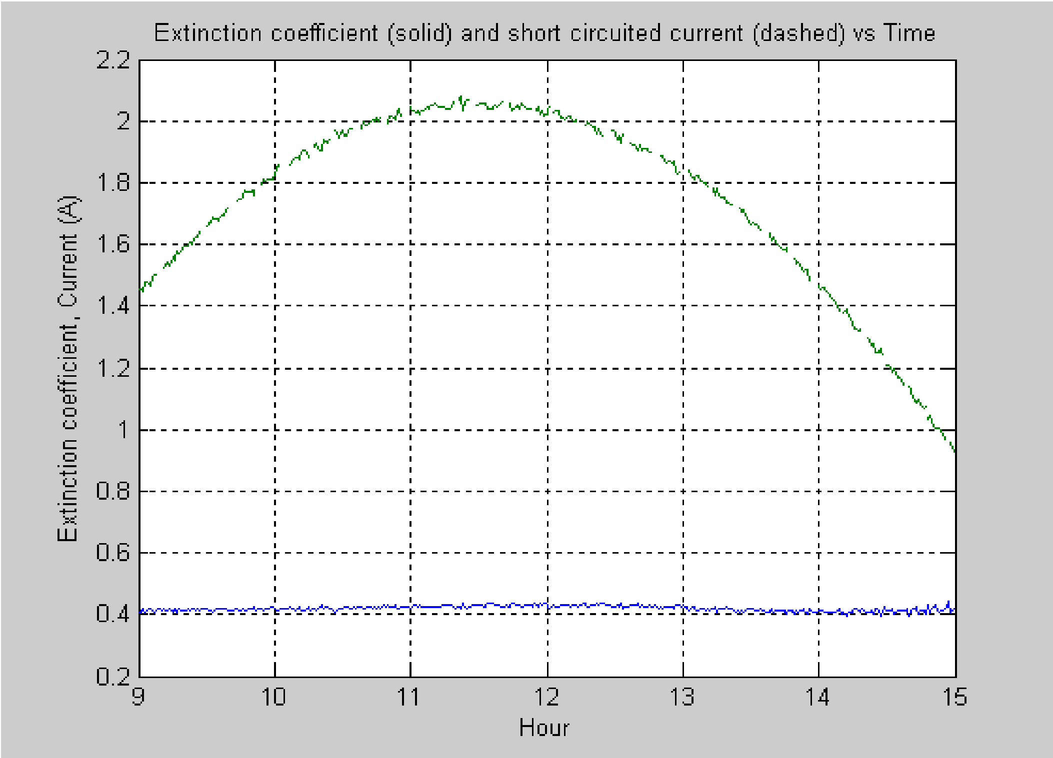

Figure 115 Extinction coefficient and short circuited current vs Time. 060719

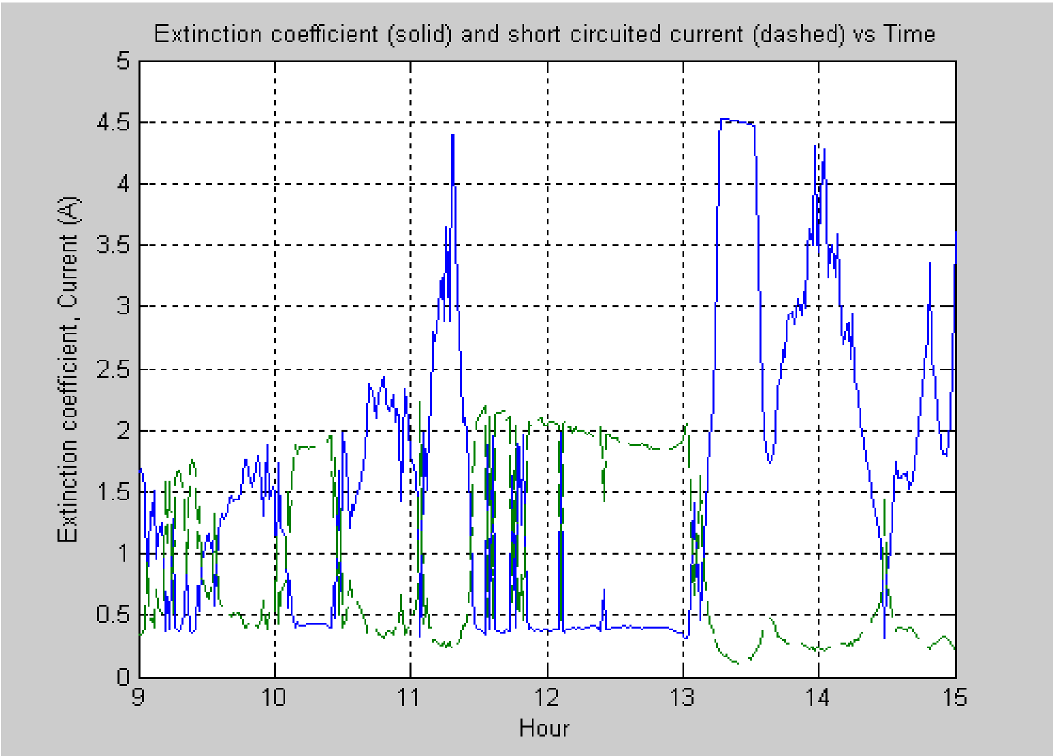

Figure 116 Extinction coefficient and short circuited current vs Time. 060622

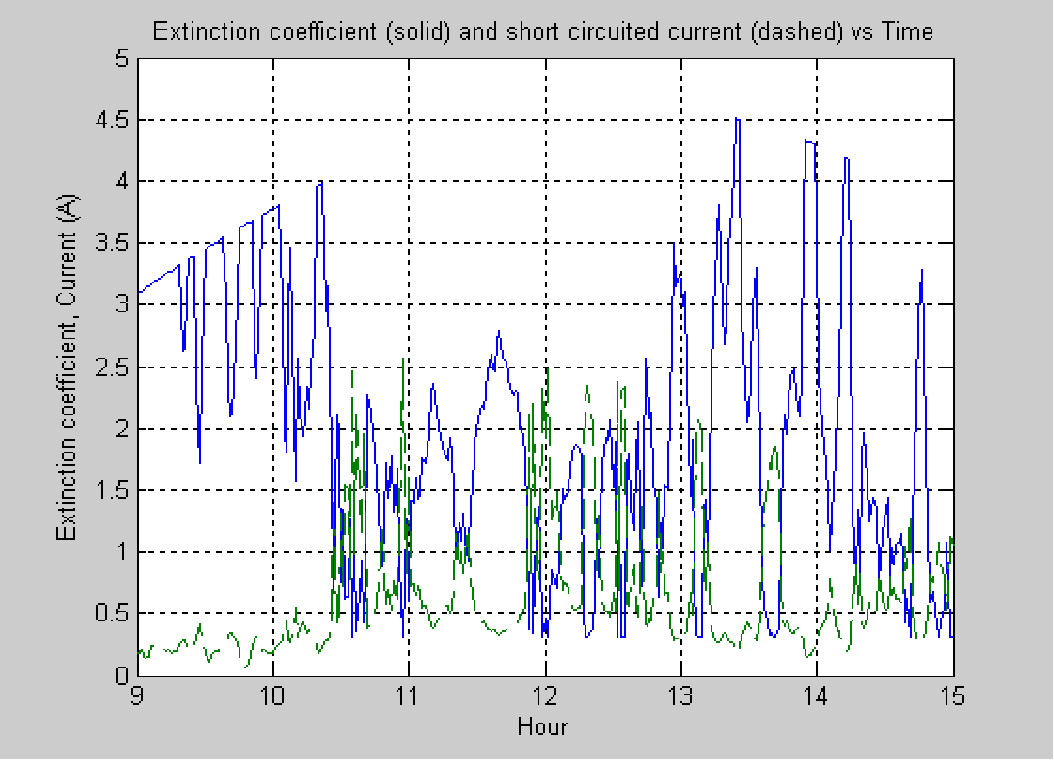

Figure 117 Extinction coefficient and short circuited current vs Time. 060623

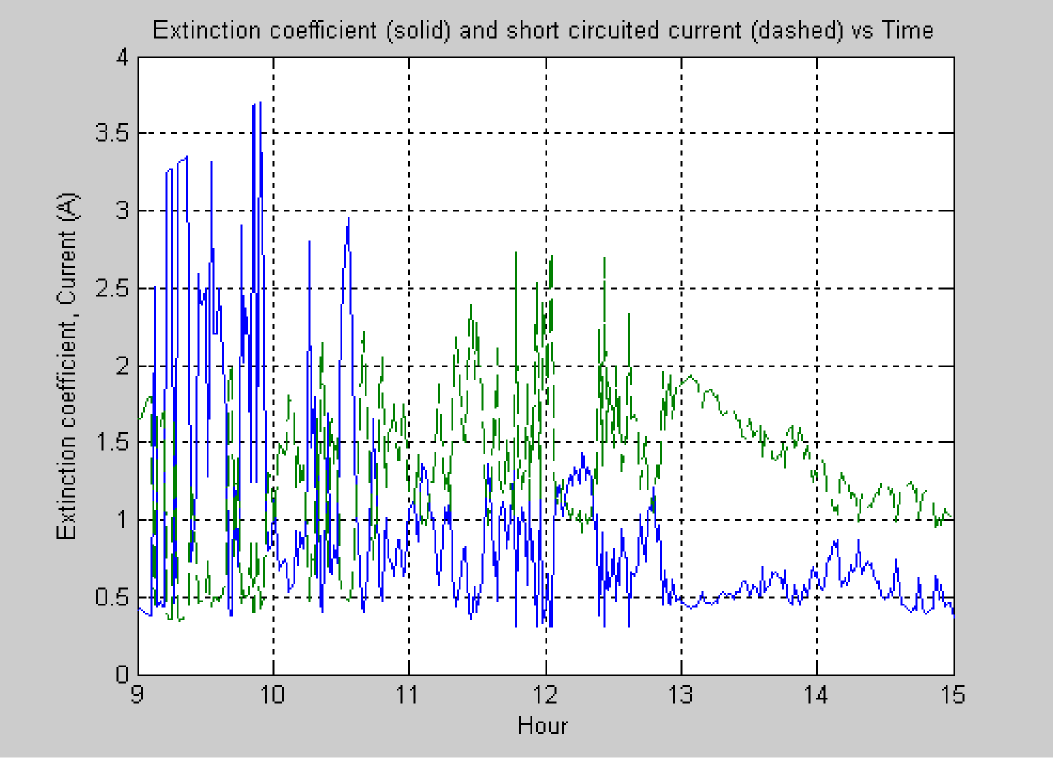

Figure 118 Extinction coefficient and short circuited current vs Time. 060624

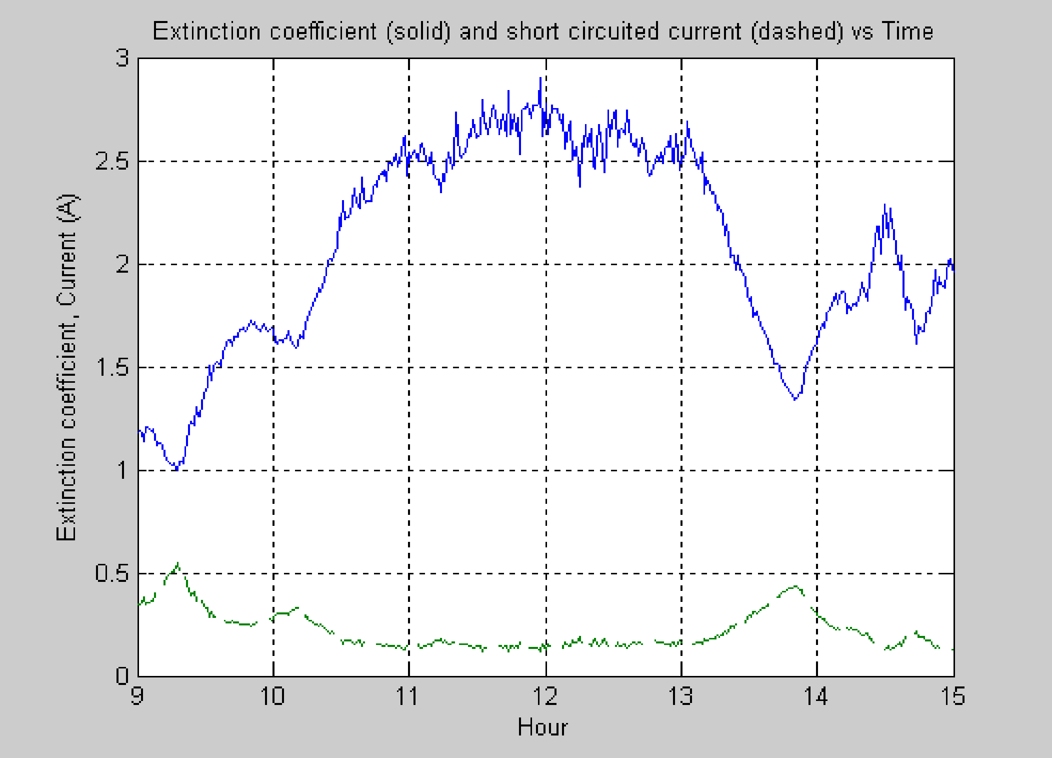

Figure 119 Extinction coefficient and short circuited current vs Time. 060626

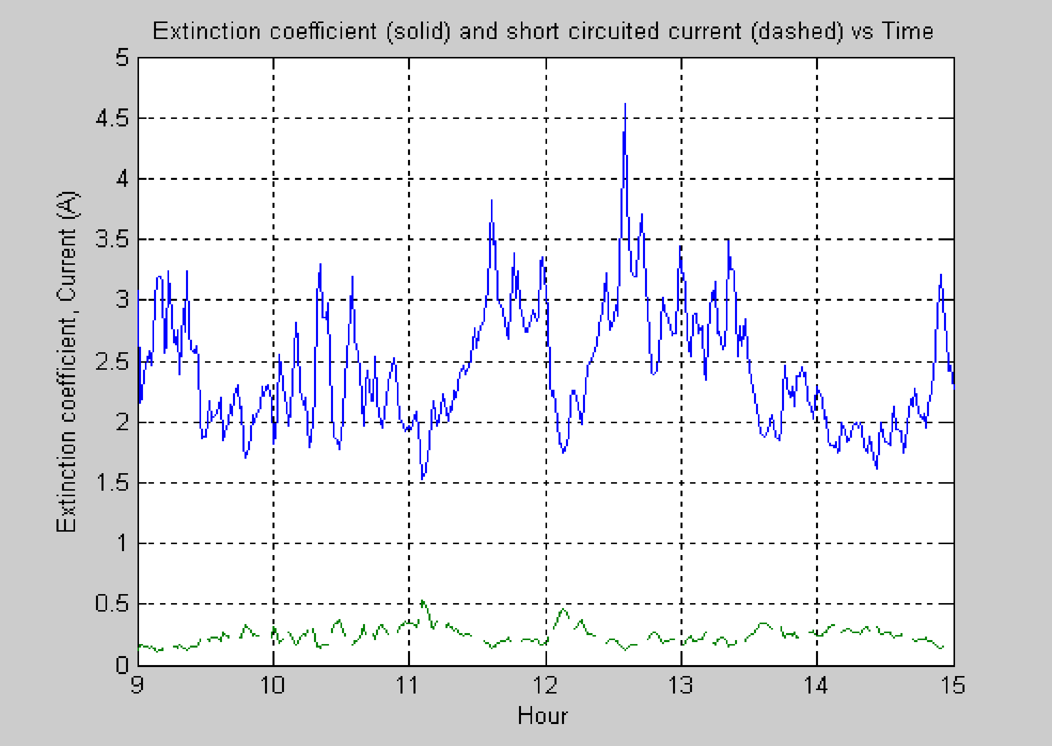

Figure 120 Extinction coefficient and short circuited current vs Time. 060627

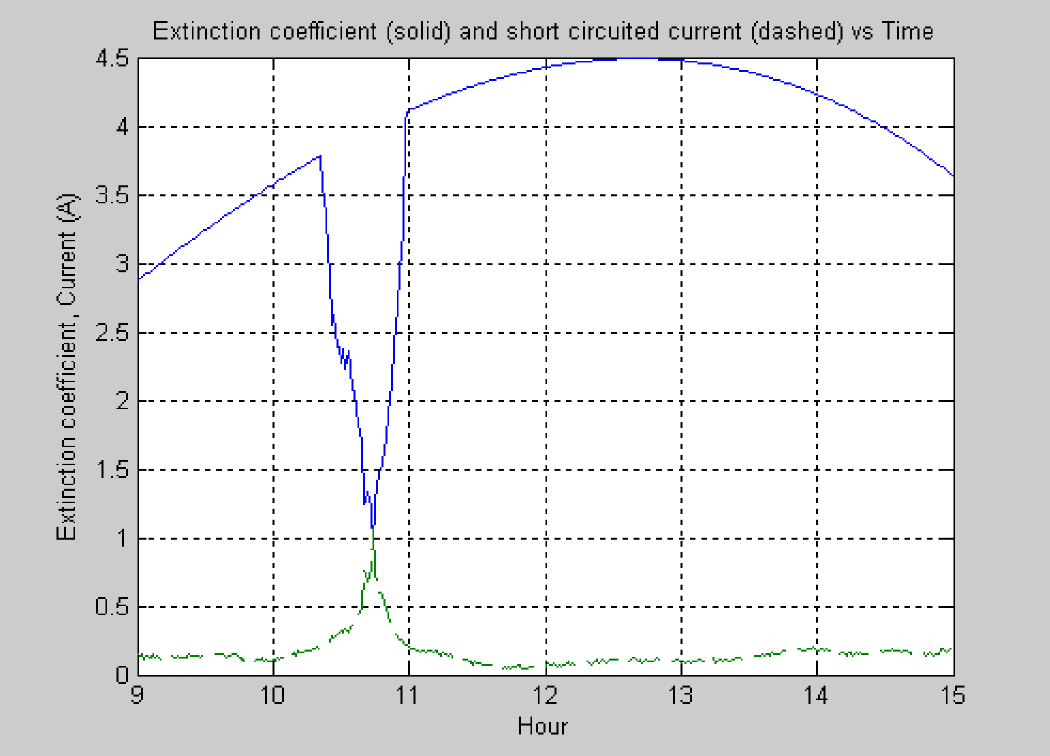

Figure 121 Extinction coefficient and short circuited current vs Time. 060721

6 REFERENCES

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